Financing Through Asset Sales*

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Abstract

Most research on firm financing studies debt versus equity issuance. We model an alternative source, non-core asset sales, and contrast it with equity. First, equity investors own a claim to the cash raised, mitigating the information asymmetry of equity (the "certainty effect"). Second, firms can disguise the sale of low-quality assets as motivated by dissynergies (the "camouflage effect"). Third, selling equity implies a "lemons" discount for not only the equity issued but also the firm, since it is perfectly correlated (the "correlation effect"). A discount on assets does not reduce the stock price since assets are not a carbon copy.

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One of the most important decisions that a firm faces is how to raise financing. Most existing research focuses on the choice between debt and equity, with various theories identifying different factors that drive a firm's security issuance decision. The pecking-order theory of Myers (1984), motivated by the model of Myers and Majluf (1984, "MM"), posits that managers issue securities that exhibit least information asymmetry. The trade-off theory argues that managers compare the benefits of debt (tax shields and a reduction in the agency costs of equity) with its costs (bankruptcy costs and the agency costs of debt). The market timing theory of Baker and Wurgler (2002) suggests that managers sell securities that are most mispriced.

While there is substantial research on financing through security issuance, another major source of financing is relatively unexplored – selling non-core assets such as a division or a plant. Asset sales are substantial in reality: Securities Data Corporation reports U.S. asset sales of $133bn in 2010, versus $130bn in seasoned equity issuance. While some of these sales may have been motivated by operational reasons, capital raising is a key driver of many others. In 2010-1, major oil and gas firms (e.g. Chevron, Shell, and Conoco) sold non-core divisions to raise capital for liquidity and debt service. Most notably, BP targeted $45bn in asset sales to cover the costs of the Deepwater Horizon disaster. Over the same period, banks worldwide raised billions of dollars via asset sales to replenish depleted capital.[1] More broadly, Hite, Owers, and Rogers (1987) examine the stated motives for asset sales and note that "in several cases, management indicated that assets were being sold to raise capital for expansion of existing lines of business or to reduce high levels of debt. In other words, selling assets was viewed as an alternative to the sale of new securities." Borisova, John, and Salotti (2011) find that over half of asset sellers state financing motives, and Hovakimian and Titman (2006) and Borisova and Brown (2012) show that asset sales lead to increases in investment and R&D, respectively, suggesting that they were undertaken to raise capital. Campello, Graham, and Harvey (2010) report that 70% of financially constrained firms increased asset sales in the financial crisis, versus 37% of unconstrained firms. Ofek (1993), Asquith, Gertner, and Scharfstein (1994), and Maksimovic and Phillips (1998) also find that firms sell assets in response to financial constraints.

This paper analyzes the role of asset sales in financing, in a model that allows for them to be undertaken not only to raise capital, but also for operational reasons (dissynergies). It studies the conditions under which asset sales are preferable to equity

[1] In September 2011, BNP Paribas and Société Générale announced plans to raise $96 billion and $5.4 billion respectively through asset sales, to create a financial buffer against contagion from other French banks. Bank of America raised $3.6 billion in August 2011 by selling a stake in a Chinese construction bank, and $755 million in November 2011 from selling its stake in Pizza Hut.

issuance and vice-versa, how financing and operational motives interact, and how firm boundaries are affected by financial constraints. We build a deliberately parsimonious model to maximize tractability; this allows for the key expressions to be solved in closed form and the economic forces to be transparent. The firm comprises a core asset and a non-core asset. The firm must raise financing to meet a liquidity need, and can sell either equity or part of the non-core asset. Following MM, we model information asymmetry as the principal driver of this choice. The firm's type is privately known to its manager and comprises two dimensions. The first is quality, which determines the assets' standalone (common) values. The value of the core asset is higher for high-quality firms. The value of the non-core asset depends on how we specify the correlation between the core and non-core assets. With a positive (negative) correlation, the value of the non-core asset is higher (lower) for high-quality firms. The second dimension is synergy: the additional value that the non-core asset is worth to its current owner.

It may seem that asset sales can already be analyzed by applying the general principles of MM's security issuance model to assets, removing the need for a new theory specific to asset sales. Such an extension would suggest that assets are preferred to equity if they exhibit less information asymmetry. While information asymmetry is indeed an important consideration, our model identifies three new forces that also drive the financing choice and may outweigh information asymmetry considerations.

First, an advantage of equity is that new shareholders obtain a stake in the entire firm. This includes not just the core and non-core assets in place (whose value is unknown), but also the cash raised (whose value is known). This mitigates the information asymmetry of the assets in place: the *certainty effect*. In contrast, an asset purchaser does not share in the cash raised, and thus bears the full information asymmetry associated with the asset's value. Hence, contrary to MM, even if equity exhibits more information asymmetry than the non-core asset, the manager may sell equity if enough cash is raised that the certainty effect dominates. Contrary to conventional wisdom, equity is not always the riskiest claim: if a large amount of financing is raised, equity becomes relatively safe.

Formally, a pooling equilibrium is sustainable where all firms sell assets (equity) if the financing need is sufficiently low (high). The choice of financing thus depends on the amount required. This dependence contrasts standard financing models, where the choice depends only on the inherent characteristics of the claim being issued (such as its information asymmetry (MM) or misvaluation (Baker and Wurgler (2002)) and not the amount required – unless one assumes exogenous limits such as notions of debt capacity. This result also has implications for the investment literature, in which disinvestment occurs due to financial constraints and so a greater financing need leads to more assets being sold. We show that a greater financial shock may reduce asset sales, as firms substitute into equity. Thus, such a shock can improve real efficiency, as firms hold onto synergistic assets and instead sell equity.

The certainty effect applies to any use of cash whose expected value is uncorrelated with firm quality: retaining it on the balance sheet to replenish capital, repaying debt, paying dividends, or financing an uncertain investment whose expected payoff is independent of firm quality. We also analyze the case in which the investment return is correlated with firm quality, and thus exhibits information asymmetry. It may appear that the certainty effect should weaken, since the funds raised are no longer held as certain cash. This intuition turns out to be incomplete, because there is a second effect. Since investment is positive-NPV, it increases the value of the capital that investors are injecting. If the desirability of investment (for firms of both quality) is high compared to the additional return generated by the high-quality firm over the low-quality firm, the second effect dominates – somewhat surprisingly, the certainty effect can strengthen when cash is used to finance an uncertain investment. This effect makes equity easier to sustain. In contrast, if the additional return generated by the high-quality firm is sufficiently large, then asset sales become preferable. Due to the role of the investment return, the source of financing depends on the use of financing. In almost all cases, it remains robust that asset (equity) sales are used for low (high) financing needs.

A second driver of the financing decision is the level of synergies between the non-core asset and the firm. This consideration leads to "threshold" semi-separating equilibria, in which a firm sells assets if synergies are below a cutoff and equity otherwise. While models of firm boundaries also predict that firms will sell dissynergistic assets, here these operational motives interact with financing / market timing reasons. Some high-quality firms sell assets not because they are low-quality, but because they are dissynergistic, allowing low-quality firms to pool with them. They can disguise an asset sale driven by overvaluation (the asset is of low quality and has a low common value) as instead being driven by operational reasons (it is dissynergistic and only has a low private value): the *camouflage effect*. Low-quality asset sellers can make greater profits than in the pooling equilibria, where the financing choice does not depend on synergies and so no disguise is possible. A market in which firms are selling assets for operational reasons is "deep" and allows other firms to exploit their private information by selling overvalued assets. This notion of "market depth" is similar to the Kyle (1985) model of securities trading, where a deep market arises when liquidity traders are selling their securities for reasons other than a low common value. Such depth allows informed traders to profit from selling securities that do have a low common value.

The threshold synergy level is different for high- and low-quality firms, due to the certainty effect. If the amount of financing required increases, this reduces the information asymmetry of equity, making it more (less) attractive to high (low) quality firms. Thus, higher financing needs have real effects. First, they affect firm boundaries by causing some firms to switch into or away from asset sales. In standard models without financial constraints, firm boundaries depend only on synergies.[2] Intuitively, adding financial constraints might suggest that divisions will be sold even if they are synergistic, due to capital needs. Our model allows firms to raise capital also through equity, and so greater capital needs may reduce asset sales as firms substitute into equity. Second, greater financing needs reduce the quality of assets traded in equilibrium (and thus their price), and increase the quality and price of equity. Thus, the market reaction to equity (asset) sales should be more (less) positive for a larger sale.

The camouflage effect continues to hold if firms have the option not to raise financing. If low-quality firms must raise financing (because their internal cash generation is low, as in Miller and Rock (1985)), but high-quality firms have a choice, we have a semi-separating equilibrium where high-quality firms with synergistic assets do nothing, and those with dissynergistic assets sell them. Low-quality firms prefer to meet their financing needs through asset sales. Issuing equity would reveal them as low-quality, since no high-quality firms do so, but asset sales allow them to disguise their financing need as being motivated by operational reasons (dissynergies) rather than desperation (low cash generation). Thus, only low-quality firms with the most synergistic assets issue equity; all others, including some with strictly positive synergies, sell assets.

A third driver is the *correlation effect*, which represents an advantage to selling assets. When a firm issues equity, it suffers an Akerlof (1970) "lemons" discount – the market infers that the equity is low-quality from the firm's decision to issue it. Not only does the market pay a low price for the equity issued, but also it attaches a low valuation to the rest of the firm, because it is perfectly correlated with the issued equity. When a firm sells non-core assets, it also receives a low price, but critically this need not imply a low valuation for the firm as the asset sold may not be a carbon copy. Thus, firms can sell poorly performing assets without sending a negative signal. Formally, in the negative correlation model, the parameter values that support the equity-pooling equilibrium are a strict subset of those that support the asset-pooling equilibrium.

[2] Synergies arise when assets are worth more together under joint ownership than separately. They may stem from transactions costs being lower within a firm than in a market (Coase (1937)), monitoring advantages (Alchian and Demsetz (1972)), economies of scope (Panzar and Willig (1981)), or addressing hold-up problems (Grossman and Hart (1986), Hart and Moore (1990)).

An implication is that conglomerates issue equity less often than firms with closely related divisions. In addition, asset sales (equity issuance) should lead to positive market reactions, as found empirically. The analysis also highlights a new benefit of diversification: a non-core asset is a form of financial slack. While the literature on investment reversibility (e.g. Abel and Eberly (1996)) models reversibility as an inherent feature of the asset's technology, here an investment that is not a carbon copy of the firm is "reversible" in that it can be sold without negative inferences.

Our paper can be interpreted more broadly as studying at what *level* to issue claims: the firm level (equity issuance) or the asset level (asset sales). Many of the effects also apply to other *types* of claim that the firm can issue at each level. All three effects apply to parent-company risky debt in the same way as parent-company equity: since debt is also a claim to the entire firm, it benefits from the certainty effect and is positively correlated with firm value; issuing debt does not involve the loss of synergies. Similarly, if a firm issues collateralized debt at the asset/division level or engages in an equity carve-out of a division, this need not imply low quality for the firm as a whole (correlation effect), and investors do not own a claim to the cash they invest, which resides at the parent company level (certainty effect).

Most existing literature on asset sales is empirical. Jain (1985), Klein (1986), Hite, Owers, and Rogers (1987), and Slovin, Sushka, and Ferraro (1995) find positive market reactions to asset sales. Lang, Poulsen, and Stulz (1995) show that this positive reaction stems from financing rather than operational reasons. Brown, James, and Mooradian (1994) and Bates (2006) examine the use of proceeds. Maksimovic and Phillips (2001) and Eisfeldt and Rampini (2006) analyze operational rather than financing motives.

Existing theories consider asset sales as the only source of financing and do not compare them to equity, e.g. Shleifer and Vishny (1992), DeMarzo (2005), He (2009), and Kurlat (2010). Milbradt (2012) and Bond and Leitner (2011) show that selling an asset will affect the market price of the seller's remaining portfolio under mark-to-market accounting. We show that such correlation effects are stronger for equity: while a partial asset sale may imply a negative valuation of the remaining unsold non-core assets, it need not imply a negative valuation of the firm. Nanda and Narayanan (1999) also consider both asset sales and equity issuance under information asymmetry, but do not feature the certainty, camouflage, or correlation effects.[3]

[3]Leland (1994) allows firms to finance cash outflows either by equity issuance (in the core analysis) or by asset sales (in an extension), but not to choose between the two. In Strebulaev (2007), asset sales are assumed to be always preferred to equity issuance, which is a last resort. Other papers model asset sales as a business decision (equivalent to disinvestment) and do not feature information asymmetry. In Morellec (2001), asset sales occur if the marginal product of the asset is less than its (exogenous)

Since a partial asset sale can be interpreted as a carve-out, our paper is also related to the carve-out literature. Nanda (1991) also notes that non-core assets may be uncorrelated with the core business and that this may motivate a firm to issue subsidiary equity. In his model, correlation is always zero and the information asymmetry of core and non-core assets is identical. Our model allows for general correlations and information asymmetries, as well as synergies, enabling us to generate the three effects.[4]

This paper is organized as follows. Section 1 outlines the general model. Sections 2 and 3 study the positive and negative correlation cases respectively. Section 4 analyzes extensions, Section 5 discusses empirical implications, and Section 6 concludes. The Appendix contains proofs and other peripheral material.

1 The Model

The model consists of two types of risk-neutral agent: firms, which raise financing, and investors, who provide financing and set prices. The firm is run by a manager, who has private information about the firm's type $\theta = (q, k)$. If a firm is of type θ, we also say that the manager is of type θ.[5] The type θ consists of two dimensions. The first is the firm's quality $q \in \{H, L\}$, which measures the standalone (common) value of its assets. The prior probability that $q = H$ is $\pi \in (\frac{1}{2}, 1)$. The second dimension is a synergy parameter $k \sim U\left[\underline{k}, \overline{k}\right]$, where $-1 < \underline{k} \leq 0$, $\overline{k} \geq 0$, and k and q are uncorrelated. This parameter measures the additional (private) value created by the existing owner.

The firm comprises two assets. The core business has value C_q, where $C_H > C_L$, and the non-core business has value A_q.[6] Where there is no ambiguity, we use the term "assets" to refer to the non-core business. We consider two specifications of the model. The first is $A_H \geq A_L$, so that the two assets are positively correlated. The second is

resale value. In Bolton, Chen, and Wang (2011), disinvestment occurs if the cost of external finance is high relative to the marginal productivity of capital. While those papers take the cost of financing as given, this paper microfounds the determinants of the cost of equity finance versus asset sales.

[4]Empirically, Allen and McConnell (1998) study how the market reaction to carve-outs depends on the use of proceeds. Schipper and Smith (1986) show that equity issuance leads to negative abnormal returns, but carve-outs lead to positive returns. Slovin, Sushka, and Ferraro (1995) find positive market reactions to carve-outs, and Slovin and Sushka (1997) study the implications of parent and subsidiary equity issuance on the stock prices of both the parent and the subsidiary.

[5]Since the manager and firm are interchangeable, we use both the personal pronoun "he" and the impersonal pronoun "its" to refer to the firm.

[6]The values C_q and A_q represent asset values net of liabilities, and so our model also incorporates information asymmetry about a firm's liabilities. For example, if a firm has unknown litigation liabilities at the parent company level, a purchaser of one of its factories is not exposed to them.

$A_L > A_H$, so the assets are negatively correlated. In both cases, we assume:

$$C_H + A_H > C_L + A_L, \quad (1)$$

i.e. H has a higher total value even if $A_H < A_L$. In MM, the key driver of financing is information asymmetry. The distinction between the two cases of $A_H \geq A_L$ and $A_H < A_L$ shows that it is not only the information asymmetry of the non-core asset that matters ($|A_H - A_L|$), but also its correlation with the core asset ($sign\,(A_H - A_L)$).

We consider an individual firm, which must raise financing of F.[7] The cash raised remains on the firm's balance sheet. This modeling treatment nests any financing need that increases equity value by an amount F in expectation, independent of q, such as replenishing capital, repaying debt, paying dividends, or financing an uncertain investment whose expected value is uncorrelated with q. In Section 4.1, the investment return is correlated with firm quality and thus exhibits information asymmetry.

We currently treat the financing need F as exogenous. In MM, the firm has the option not to raise financing and instead to forgo investment; their goal was to show that information asymmetry can deter investment by hindering financing. Since this effect is already well-known, our focus instead is the choice between asset sales and equity to meet a given financing need. In Section 4.2, we give firms the choice of whether to raise financing and allow financing needs to be privately known.

The firm can raise F by selling either non-core assets or equity; it cannot sell the core asset as it is essential to the firm. (In Appendix B, we relax this assumption.) We specify $F \leq \min(A_L, A_H)$, so that the financing can be raised entirely through either source.[8] We assume that the firm uses a single source. This can be motivated by the transactions costs of using multiple sources. There are no taxes, and any transactions costs are the same for both sources. Firms cannot raise financing in excess of the requirement F; this assumption can be justified by forces outside the model such as agency costs of free cash flow.

The non-core asset is perfectly divisible so partial asset sales are possible. We do not feature nonlinearities as they will mechanically lead to the source of financing depending on the amount required. If a firm sells non-core assets worth Y, its fundamental value

[7] The amount of financing F does not depend on the source of financing: F must be raised regardless of whether the firm sells assets or equity. In bank capital regulation, equity issuance leads to a superior improvement in capital ratios than asset sales and so F does depend on the source of financing. We do not consider this effect here as the effect will be straightforward: it will encourage H towards the source that reduces the amount of financing required, and thus force L to follow in order to pool.

[8] Some of the analysis in the paper will derive bounds on F for various equilibria to be satisfied. We have verified that none of these bounds are inconsistent with $F < \min(A_L, A_H)$.

falls by $Y(1+k)$. Thus, the case of $k > (<) 0$ represents synergies (dissynergies), where the asset is worth more (less) to the current owner than a potential purchaser. That $\underline{k} \leq 0$ allows for asset sales to be motivated by operational reasons (dissynergies) rather than only financing reasons.[9] In addition to synergies, $k > 0$ can also arise if investment in assets is costly to reverse (e.g. Abel and Eberly (1996)).

Formally, a firm of type θ issues a claim $X \in \{E, A\}$, where $X = E$ represents equity and $X = A$ assets. Investors infer θ based on X. These inferences affect both the firm's stock price and the terms of financing (and thus fundamental value). Investors are perfectly competitive and price both the claim being sold and the firm's stock at their expected values conditional upon X.[10] The manager maximizes firm value; in the negative correlation case of Section 3, he also cares about the stock price.

A useful feature of the framework is that only quality q, and not synergy k, directly affects the investor's valuation of a claim and thus the price paid. This allows our model to incorporate two dimensions of firm type – quality and synergy – while retaining tractability. We sometimes use the term "H" or "H-firm" to refer to a high-quality firm regardless of its synergy parameter, and similarly for "L" or "L-firm". "Capital gain/loss" refers to the gain/loss resulting from the common value component of the asset value only, and "fundamental gain/loss" refers to the change in the firm's overall value, which consists of both the capital gain/loss and any loss of (dis)synergies. For equity issuance, the capital gain/loss equals the fundamental gain/loss.

We solve for pure strategy equilibria.[11] We use the Perfect Bayesian Equilibrium ("PBE") solution concept, where: (i) Investors have a belief about which types issue which claim X; (ii) The price of the claim being issued equals its expected value, conditional on investors' beliefs in (i); (iii) Each manager type chooses to issue the claim X that maximizes his objective function, given investors' beliefs; (iv) Investors' beliefs satisfy Bayes' rule. In addition to the PBE, beliefs on claims X issued off the equilibrium path satisfy the Cho and Kreps (1987) Intuitive Criterion ("IC").

[9] One may wonder why the firm will have dissynergistic assets to begin with. Firms may acquire assets when they are synergistic, but they may become dissynergistic over time. One may still wonder why the firm has not yet disposed of the dissynergistic asset. First, the firm may retain it due to the transactions costs of asset sales: only if it is forced to raise financing and so would have to bear the transactions costs of equity issuance otherwise, would it consider selling assets. Second, the market for assets is not perfectly frictionless, and so not all assets are owned by the best owner at all times. Our model allows for $\underline{k} = 0$ in which case there are no dissynergies; more generally, our model specifies that synergies are not so strong as to overwhelm the other forces in the model.

[10] Sicherman and Pettway (1987), John and Ofek (1995), and Datta, Iskandar-Datta, and Raman (2003) find insignificant returns to purchasers in asset sales.

[11] Mixed strategy equilibria only exist for the type that is exactly indifferent between the two claims. Since synergies are continuous, this type is atomistic and so it does not matter for posterior beliefs whether we specify this cutoff type as mixing or playing a pure strategy.

We first analyze the positive correlation version of the model ($A_H \geq A_L$) and then move to negative correlation ($A_L > A_H$).

2 Positive Correlation

We first consider pooling equilibria (PE), which are of two types: an asset-pooling equilibrium (APE) and an equity-pooling equilibrium (EPE). We then move to semi-separating equilibria (SE). The analysis studies the conditions under which the different equilibria are sustainable, to predict when firms will use each financing channel.

2.1 Pooling Equilibrium, All Firms Sell Assets

We consider a PE in which all firms sell assets, supported by the off-equilibrium path belief (OEPB) that an equity issuer is of type $\left(L, \overline{k}\right)$. Assets are valued at

$$\mathbb{E}\left[A\right] = \pi A_H + (1-\pi)A_L. \tag{2}$$

If equity is sold (off the equilibrium path), it is valued at E_L, where

$$E_q = C_q + A_q + F$$

is the value of equity for a firm of quality q. The F term arises because the cash raised enters the balance sheet, and so new shareholders own a claim to it.[12]

The fundamental values of H and L are respectively given by:

$$C_H + A_H - F\frac{(1-\pi)\left(A_H - A_L\right) + kA_H}{\mathbb{E}\left[A\right]}, \tag{3}$$

$$C_L + A_L + F\frac{\pi(A_H - A_L) - kA_L}{\mathbb{E}\left[A\right]}. \tag{4}$$

An L-firm enjoys a capital gain of $F\frac{\pi(A_H - A_L)}{\mathbb{E}[A]}$ by selling low-quality assets at a pooled price. However, it loses the synergies from the asset. If

$$1 + \overline{k} \leq \frac{\mathbb{E}[A]}{A_L}, \tag{5}$$

then even the L-firm with the greatest synergies, type $\left(L, \overline{k}\right)$, is willing to sell assets,

[12] This is consistent with the treatment of financing in MM, although the level of financing plays no role in their analysis since both equity and debt are claims on the entire balance sheet.

since the capital gain exceeds the synergy loss. If (5) is violated, synergies are sufficiently high that $(L, \overline{k})$ will not sell assets, despite the capital gain, and so APE cannot hold. Equation (5) is necessary and sufficient for all L-firms not to deviate.

H-firms suffer a capital loss of $F\frac{(1-\pi)(A_H-A_L)}{\mathbb{E}[A]}$ in addition to any loss of synergies, and thus may deviate to equity. If they do so, fundamental value becomes:

$$C_H + A_H - F\frac{C_H - C_L + A_H - A_L}{C_L + A_L + F}. \tag{6}$$

The no-deviation ("ND") condition is that (6) $\leq$ (3). This condition is most stringent for type $(H, \overline{k})$. Thus, no H-firms will deviate if:

$$F \leq F^{APE,ND,H} = \frac{\mathbb{E}[A](C_H + A_H) - A_H(C_L + A_L)(1 + \overline{k})}{A_H(1 + \overline{k}) - \mathbb{E}[A]}. \tag{7}$$

Condition (7) is equivalent to the "unit cost of financing" being lower for assets, i.e.

$$\frac{A_H\left(1 + \overline{k}\right)}{\mathbb{E}[A]} \leq \frac{C_H + A_H + F}{C_L + A_L + F}, \tag{8}$$

where the numerator on each side is the value of the claim being sold to the firm, and the denominator is the price that investors pay for that claim.

There are three forces that determine $(H, \overline{k})$'s incentives to deviate. The first is whether assets or equity exhibit greater information asymmetry ($\frac{A_H}{\mathbb{E}[A]}$ versus $\frac{C_H+A_H}{C_L+A_L}$). This effect is a natural extension of the MM principle that high-quality firms wish to issue safe claims. Indeed, without synergies ($\overline{k} = 0$), then if $\frac{A_H}{\mathbb{E}[A]} > \frac{C_H+A_H}{C_L+A_L}$, i.e. assets exhibit sufficiently greater information asymmetry, H will deviate to equity: for any F, (8) is violated and so APE is unsustainable.

The second force is synergies, which are absent in MM. For APE to hold, firms must be willing to sell assets despite the loss of synergies. Thus, we require not only assets to be safe, but also the maximum synergy level to be small. If $1 + \overline{k} > \frac{(C_H+A_H)\mathbb{E}[A]}{(C_L+A_L)A_H}$, then again (8) is violated and so APE is unsustainable for any F.

The third force is the amount of financing F. This is unique to a model of asset sales and arises because an equity investor has a claim to the cash raised but an asset purchaser does not. Since the value of cash is certain, it mitigates the information asymmetry of equity: the right-hand side ("RHS") of (8) becomes dominated by the term F, which is the same in the numerator and the denominator as it is known, and less dominated by the unknown assets-in-place terms C_q and A_q. Thus, there is an upper bound on F to prevent deviation, given by (7). If F exceeds this bound, this "certainty effect" is sufficiently strong that $\left(H,\overline{k}\right)$ deviates to equity. In particular, even if $\frac{A_H}{\mathbb{E}[A]} < \frac{C_H+A_H}{C_L+A_L}$ and $\overline{k} = 0$, i.e. assets are safer than equity and there are no synergies, a high F can lead to (8) being violated. Thus, the MM result that firms issue the claim with the least information asymmetry does not hold. Similarly, the analysis contradicts the conventional wisdom that equity is the riskiest claim. If the amount of financing raised is sufficiently large, equity is relatively safe.

We now verify whether the OEPB, that an equity issuer is of type $\left(L,\overline{k}\right)$, satisfies the IC. This is the case if $\left(L,\overline{k}\right)$ would issue equity if inferred as H, which occurs if:

$$F \leq F^{APE,IC} = \frac{A_L(C_H + A_H)(1+\overline{k}) - \mathbb{E}\left[A\right](C_L + A_L)}{\mathbb{E}[A] - A_L(1+\overline{k})}. \quad (9)$$

It may seem that the IC should be trivial, since deviation leads to a high price for selling equity rather than a pooled price for selling assets. However, if F is large, selling equity is less attractive since the certainty effect reduces the gains from being inferred as H. Thus, we have another upper bound on F. If $1+\overline{k} < \frac{(C_L+A_L)\mathbb{E}[A]}{(C_H+A_H)A_L}$, assets exhibit relatively high information asymmetry and synergies are small. Thus, $\left(L,\overline{k}\right)$ enjoys such a large fundamental gain from asset sales that he will not deviate to equity even if revealed as H: the RHS of (9) is negative and so the IC is violated for any F.[13]

Lemma 1 below summarizes the equilibrium. The proof shows that, if and only if $1+\overline{k} < \frac{\mathbb{E}[A]}{\sqrt{A_H A_L}}$ (>1), the IC condition is stronger than the ND condition and thus is the relevant condition for APE to hold. (All proofs are in Appendix A).

Lemma 1. *(Positive correlation, pooling equilibrium, all firms sell assets.) Consider a pooling equilibrium where all firms sell assets ($X = A$) and a firm that issues equity is inferred as type $\left(L,\overline{k}\right)$. The prices of assets and equity are $\pi A_H + (1-\pi)A_L$ and $C_L + A_L + F$ respectively. The equilibrium is sustainable if the following conditions hold:*

(i) $1+\overline{k} \leq \frac{\mathbb{E}[A]}{A_L}$,

(ii) $F \leq F^{APE}$, *where*

$$F^{APE} = \begin{cases} F^{APE,IC} \equiv \frac{A_L(C_H+A_H)(1+\overline{k}) - \mathbb{E}[A](C_L+A_L)}{\mathbb{E}[A]-A_L(1+\overline{k})} & \text{if } 1+\overline{k} \leq \frac{\mathbb{E}[A]}{\sqrt{A_H A_L}} \\ F^{APE,ND,H} \equiv \frac{\mathbb{E}[A](C_H+A_H)-A_H(C_L+A_L)(1+\overline{k})}{A_H(1+\overline{k})-\mathbb{E}[A]} & \text{if } 1+\overline{k} \geq \frac{\mathbb{E}[A]}{\sqrt{A_H A_L}}. \end{cases} \quad (10)$$

[13] To eliminate an equilibrium with $F > F^{APE,IC}$ via the IC, we need to show that the only reasonable OEPB is that a deviator is of quality H, which also requires us to show that some firm of quality H will deviate if revealed good. This will automatically be the case, as $(H, 0)$ will break even rather than suffering a capital loss and losing synergies. In all of the other equilibria that we consider, it will similarly be automatic that $(H, 0)$ will deviate if he is revealed good, so we will not need to show this mathematically.

2.2 Pooling Equilibrium, All Firms Sell Equity

We now consider the alternative PE in which all firms issue equity, supported by the OEPB that an asset seller is of type $(L, \underline{k})$. Equity is valued at

$$\mathbb{E}\left[E\right] = \pi\left(C_H + A_H\right) + \left(1 - \pi\right)\left(C_L + A_L\right) + F$$

and if assets are sold (off the equilibrium path), they are valued at A_L.

As in APE, L makes a capital gain; however, he will deviate to assets if they are sufficiently dissynergistic. Type $(L, \underline{k})$ has the greatest incentive to deviate. His ND condition is given by $1 + \underline{k} \geq \frac{E_L}{\mathbb{E}[E]}$, which can be rewritten

$$F \leq F^{EPE,ND,L} = \frac{\mathbb{E}[C + A](1 + \underline{k}) - (C_L + A_L)}{-\underline{k}}. \tag{11}$$

H-firms will not deviate if:

$$F \geq F^{EPE,ND,H} = \frac{A_L(C_H + A_H) - A_H\mathbb{E}[C + A](1 + \underline{k})}{A_H(1 + \underline{k}) - A_L} \tag{12}$$

and $1 + \underline{k} > \frac{A_L}{A_H}$. If $1 + \underline{k} < \frac{A_L}{A_H}$, the inequality in (12) would change sign and F would have to be less than a negative number. Intuitively, if dissynergies are too strong, $(H, \underline{k})$ will deviate to asset sales. In contrast to Section 2.1, H's ND condition (12) now imposes a *lower* bound on F. This also results from the certainty effect. If F is high, H suffers a small loss from equity issuance, and so will not deviate.

The OEPB, that an asset seller is of type $(L, \underline{k})$, satisfies the IC if

$$F \geq F^{EPE,IC} = \frac{A_L\mathbb{E}[C + A](1 + \underline{k}) - A_H(C_L + A_L)}{A_H - A_L(1 + \underline{k})}. \tag{13}$$

The denominator is positive, and so the lower bound can always be satisfied for some F: unlike in APE, there is no necessary condition.

Lemma 2 below summarizes the equilibrium.

Lemma 2. *(Positive correlation, pooling equilibrium, all firms sell equity.) Consider a pooling equilibrium where all firms sell equity ($X = E$) and a firm that sells assets is inferred as type $(L, \underline{k})$. The prices of assets and equity are A_L and $\pi\left(C_H + A_H\right) + \left(1 - \pi\right)\left(C_L + A_L\right) + F$ respectively. This equilibrium is sustainable if the following conditions hold:*

(i) $1 + \underline{k} > \max\left(\frac{A_L}{A_H}, \frac{E_L}{\mathbb{E}[E]}\right)$,

(ii) $F \geq F^{EPE}$*, where*

$$F^{EPE} = \begin{cases} F^{EPE,IC} \equiv \frac{A_L \mathbb{E}[C+A](1+\underline{k}) - A_H(C_L+A_L)}{A_H - A_L(1+\underline{k})} & \textit{if } 1+\underline{k} \geq \frac{A_H A_L}{\mathbb{E}[A^2]} \\ F^{EPE,ND,H} \equiv \frac{A_L(C_H+A_H) - A_H \mathbb{E}[C+A](1+\underline{k})}{A_H(1+\underline{k}) - A_L} & \textit{if } 1+\underline{k} \leq \frac{A_H A_L}{\mathbb{E}[A^2]}. \end{cases} \quad (14)$$

2.3 Semi-Separating Equilibria

In a SE, the financing choice depends on the synergy parameter k: there is a cutoff k_q^* where any firm below (above) the cutoff sells assets (equity). In this subsection we thus assume that $\overline{k}$ is strictly greater than $\underline{k}$: in the limit case of $\overline{k} = \underline{k} = 0$, there is no synergy parameter to separate along. H and L can use different cutoff rules, so separation will be along both type dimensions.

While investors do not directly care about k (as it only affects private values), the synergy cutoffs affect the expected quality (common value) of the claims. Using Bayes' rule, the prices paid for sold assets and issued equity are, respectively:

$$\mathbb{E}[A|X=A] = \pi \frac{k_H^* - \underline{k}}{\mathbb{E}[k_q^*] - \underline{k}} A_H + (1-\pi) \frac{k_L^* - \underline{k}}{\mathbb{E}[k_q^*] - \underline{k}} A_L, \quad (15)$$

$$\mathbb{E}[E|X=E] = \pi \left(\frac{\overline{k} - k_H^*}{\overline{k} - \mathbb{E}\left[k_q^*\right]} \right) (C_H + A_H) + (1-\pi) \left(\frac{\overline{k} - k_L^*}{\overline{k} - \mathbb{E}\left[k_q^*\right]} \right) (C_L + A_L) + F, \quad (16)$$

where

$$\mathbb{E}\left[k_q^*\right] = \pi k_H^* + (1-\pi) k_L^*.$$

A type (q, k) will prefer equity if and only if its unit cost of financing is no greater:

$$\frac{C_q + A_q + F}{\mathbb{E}[E|X=E]} \leq \frac{A_q(1+k)}{\mathbb{E}[A|X=A]}, \quad (17)$$

The cutoff k_q^* is that which allows (17) to hold with equality. Thus, it is defined by:

$$k_q^* = \frac{C_q + A_q + F}{A_q} \frac{\mathbb{E}[A|X=A]}{\mathbb{E}[E|X=E]} - 1. \quad (18)$$

Although k_q^* is not attainable in closed form, we can study whether $k_H^* \lesseqgtr k_L^*$. Since only the $\frac{C_q + A_q + F}{A_q}$ term on the RHS depends on q, the higher cutoff k_q^* will belong to the quality q for which this term is higher. Thus, $k_H^* > k_L^*$ if and only if

$$\frac{C_H + A_H + F}{C_L + A_L + F} > \frac{A_H}{A_L} \tag{19}$$

$$F < F^* \equiv \frac{C_H A_L - C_L A_H}{A_H - A_L}. \tag{20}$$

Condition (19) is intuitive. It requires the certainty effect-adjusted information asymmetry to be higher for equity, which in turn requires F to be low. H dislikes information asymmetry as it increases his capital loss; conversely, L likes information asymmetry. Thus, if F falls, equity becomes less (more) attractive to H (L); therefore, the threshold synergy below which H sells assets is higher.

The different cutoffs in turn affect the valuations. If $k_H^* > k_L^*$, H is more willing to sell assets than L. Thus, asset sales are a positive signal of quality, and so the asset price (15) is higher than in the APE (2). As a result, L-firms who sell assets make an even greater capital gain. Their sale of assets is motivated by overvaluation: the assets are low-quality and have a low common value. However, they are able to disguise the sale as instead being motivated by operational reasons, by pooling with H-firms who are indeed selling for operational reasons (the assets are dissynergistic and thus have a low private value, but are high-quality and have a high common value). Thus, they receive a higher price than under APE, where financing choices are independent of synergies and no such disguise is possible. We call this the "camouflage effect".

The camouflage effect interacts with the certainty effect. The amount required F changes the cutoffs and thus the quality of assets and equity sold in equilibrium, in turn affecting their prices. If $F > F^*$, (19) is violated: the certainty effect is sufficiently strong that equity is more attractive to H ($k_H^* < k_L^*$). More H firms sell equity, increasing (decreasing) the quality and price of equity (assets) sold. Indeed, when $F > F^*$, we have $k_H^* < 0$: since assets exhibit greater information asymmetry, H will retain them even if they are mildly dissynergistic (and when $F < F^*$, we have $k_H^* > 0$, so H will sell assets even if they are mildly synergistic). Now, L-firms achieve camouflage by selling equity: they pool with H-firms who issue equity not because it is of low quality, but because they do not wish to part with synergistic assets.

The above results are summarized in Lemma 3 below, which also gives necessary and sufficient conditions for a SE to hold.

Lemma 3. *(Positive correlation, semi-separating equilibrium): Consider a semi-separating equilibrium where quality q sells assets if $k \leq k_q^*$ and sells equity if $k > k_q^*$, where k_q^* is defined by (18). We have the following cases:*

(ia) If $F < F^*$, then $k_H^* > 0$ and $k_H^* > k_L^*$.

(ib) If $F > F^*$, then $k_H^* < 0$ and $k_H^* < k_L^*$.

(ic) If $F = F^*$, then $k_L^* = k_H^* = 0$.

The prices of assets and equity are given by (15) and (16) respectively.

A full semi-separating equilibrium where both qualities q strictly separate ($\underline{k} < k_q^* < \overline{k}$ so that both cutoffs are interior) is sustainable under the following conditions:

(iia) If $F < F^*$, a necessary condition is $1 + \overline{k} > \frac{E_H}{A_H} \frac{E[A]}{E[E]}$ and a sufficient condition is $1 + \overline{k} \geq \frac{E_H}{E_L}$.

(iib) If $F > F^*$, a necessary condition is $1 + \underline{k} < \frac{E_H}{A_H} \frac{E[A]}{E[E]}$ and a sufficient condition is $1 + \underline{k} \leq \frac{A_L}{A_H}$.

(iic) If $F = F^*$, this is sufficient for existence.

A partial semi-separating equilibrium where H's cutoff is at a boundary is sustainable in the following cases:

(iiia) If $F < F^*$, a SE where all H-firms sell assets ($k_H^* = \overline{k}$) and L-firms strictly separate ($\underline{k} < k_L^* < \overline{k}$) is sustainable only if $\frac{\mathbb{E}[A]}{A_L} < 1 + \overline{k} < \frac{E_H}{E_L}$ and if $\frac{A_H}{A_L} \leq 1 + \overline{k} \leq \frac{\mathbb{E}[A]}{A_H} \frac{E_H}{E_L}$. In this SE we have $k_L^* > 0$.

(iiib) If $F > F^*$, a SE where all H-firms sell equity ($k_H^* = \underline{k}$) and L-firms strictly separate ($\underline{k} < k_L^* < \overline{k}$) is sustainable only if $\frac{A_L}{A_H} < 1 + \underline{k} < \frac{E_L}{\mathbb{E}[E]}$ and if $\frac{A_L}{A_H} \frac{E_H}{\mathbb{E}[E]} \leq 1 + \underline{k} \leq \frac{E_L}{E_H}$. In this SE we have $k_L^* < 0$.

Lemma 3 shows that it is the relative importance of operational motives (determined by the absolute values of $\overline{k}$ and $\underline{k}$) compared to certainty effect-adjusted information asymmetry (determined by the distance of F from F^*) that governs whether SE is sustainable. In a SE, both claims are issued and one claim will be associated more with L. If F is extreme (very low or very high), information asymmetry is strong, and so issuing the claim associated with L leads to a large capital loss. If synergies are too weak to offset this loss, firms pool. In contrast, if F close to F^* and $\overline{k}$ or $\underline{k}$ is extreme, synergy motives are strong relative to information asymmetry, and so firms of the same quality issue different claims depending on k. We thus have a full semi-separating equilibrium (FSE), where firms of both quality separate. For example, if $F < F^*$, we need $1 + \overline{k} > \frac{E_H}{A_H} \frac{\mathbb{E}[A]}{\mathbb{E}[E]}$, which requires high $\overline{k}$ (strong synergies) and high F (while still satisfying $F < F^*$, so F close to F^*) so that $\frac{E_H}{\mathbb{E}[E]}$ on the RHS is low via the certainty effect. In the intermediate case, where synergies are moderate relative to information asymmetry, we have a partial semi-separating equilibrium (PSE) where all H-firms issue the same claim, regardless of k, and L-firms strictly separate. Regardless of whether we have a FSE or PSE, it remains the case that if $F > F^*$ (the certainty effect is strong), we have $k_H^* < 0$ and $k_H^* < k_L^*$ (H prefers equity); if $F < F^*$ (the certainty effect is weak), we have $k_H^* > 0$ and $k_H^* > k_L^*$ (H prefers assets).

Appendix C shows that, if synergies are extreme, a PSE is sustainable where all L-firms issue the same claim and H-firms strictly separate. This equilibrium requires synergies to be so strong that they swamp information asymmetry, and so no L-firm deviates even though it would be inferred as H. Since the paper considers the trade-off between information asymmetry and synergies, and this case requires synergies to be so strong that they dominate the trade-off, we defer the analysis to an Appendix. The intuition behind these equilibria are similar to the PSEs considered in Lemma 3.

2.4 Comparing the Equilibria

We now compare the sufficient conditions for each equilibrium to be sustainable. The results are given in Proposition 1 below:

Proposition 1. *(Positive correlation, comparison of equilibria.)*

(i) If $1 + \overline{k} \leq \frac{\mathbb{E}[A]}{A_L}$ *or* $1 + \underline{k} > \max\left(\frac{A_L}{A_H}, \frac{E_L}{\mathbb{E}[E]}\right)$*, at least one pooling equilibrium is sustainable. If both inequalities hold:*

(ia) An asset-pooling equilibrium is sustainable if $F \leq F^{APE}$*.*

(ib) An equity-pooling equilibrium is sustainable if $F \geq F^{EPE}$*.*

(ic) $F^{APE} \geq F^{EPE}$*. For* $F^{EPE} \leq F \leq F^{APE}$*, both pooling equilibria are sustainable.*

(ii) If $\frac{\mathbb{E}[A]}{A_H}\frac{E_H}{E_L} \geq 1 + \overline{k} \geq \frac{A_H}{A_L} (> \frac{\mathbb{E}[A]}{A_L})$*, a partial semi-separating equilibrium where* H *pools on assets is sustainable. The upper bound* $\frac{\mathbb{E}[A]}{A_H}\frac{E_H}{E_L} \geq 1 + \overline{k}$ *is equivalent to* $F \leq F^{APE,ND,H}$*.*

(iii) If $\frac{A_L}{A_H}\frac{E_H}{\mathbb{E}[E]} \leq 1 + \underline{k} \leq \frac{E_L}{E_H} (< \frac{E_L}{\mathbb{E}[E]})$*, a partial semi-separating equilibrium where* H *pools on equity is sustainable. The lower bound* $\frac{A_L}{A_H}\frac{E_H}{\mathbb{E}[E]} \leq 1 + \underline{k}$ *is equivalent to* $F \geq F^{EPE,ND,H}$*.*

(iv) If $1 + \overline{k} \geq \frac{E_H}{E_L} (> \frac{\mathbb{E}[A]}{A_H}\frac{E_H}{E_L})$*, a full semi-separating equilibrium where* $k_H^* > 0$ *is sustainable if* $F < F^*$*.*

(v) If $1 + \underline{k} \leq \frac{A_L}{A_H} (< \frac{A_L}{A_H}\frac{E_H}{\mathbb{E}[E]})$*, a full semi-separating equilibrium where* $k_H^* < 0$ *is sustainable if* $F > F^*$*.*

Part (i) of Proposition 1 states that pooling equilibria are sustainable if synergies are weak. Intuitively, deviation from a PE leads to being inferred as L; if synergies are not strong enough to outweigh the capital loss, deviation is ruled out and so the PE holds. When the amount of financing required increases, firms switch from selling assets (APE) to issuing equity (EPE), since the certainty effect strengthens.[14] Thus,

[14]When F becomes too high ($F > F^{EPE,ND,L}$), then $1 + \underline{k} > \frac{E_L}{\mathbb{E}[E]}$ no longer holds and EPE is

the type of claim issued depends not only on the inherent characteristics of the claim (its information asymmetry and synergies) but also the amount of financing required. In standard theories, the type of security issued only depends on its characteristics (information asymmetry or overvaluation), unless one assumes nonlinearities such as limited debt capacity. Here, F can be fully raised by either source.

It may seem that, since financing is a motive for asset sales, greater financing needs should lead to more asset sales. This result is also delivered by investment models where financial constraints induce disinvestment or reduce investment. However, here, if F rises sufficiently, the firm may sell *fewer* assets, since it substitutes into an alternative source of financing: equity. Surprisingly, greater financial constraints may improve real efficiency as firms hold onto their synergistic assets.[15]

One interesting case is a single-segment firm, which corresponds to $C_q = A_q$, i.e. core and non-core assets are one and the same. Since the information asymmetry of the firm equals that of the non-core asset, the certainty effect will push the information asymmetry of equity lower, and so lead to a preference for equity.

Parts (ii) and (iii) show that, if synergies are moderate, partial semi-separating equilibria may hold. Just as in the PEs, all H-firms sell assets (equity) for low (high) F. The difference is that, now, some L-firms are willing to deviate. Even though they are revealed as low-quality, this is outweighed by the synergy motives.

Parts (iv) and (v) show that full semi-separating equilibria may hold if synergies are strong *relative* to information asymmetry. Even if the conditions in part (i) hold (synergies are weak and so PEs are sustainable), a FSE may also be sustainable if F is close to F^* so that information asymmetry is also weak.[16] Regardless of the equilibrium (PE, FSE, or PSE), it remains robust that H prefers assets (equity) for low (high) F, because the certainty effect reduces the information asymmetry of equity.

Combining all parts of Proposition 1 together, if we fix synergies such that $1 + \overline{k} < \frac{\mathbb{E}[A]}{A_L}$, as F rises, we move from an APE, to a region in which both PEs hold, then to EPE, and finally to a PSE where all H-firms sell equity (when F becomes very high, L-firms make little capital gain from equity, and so those with highly dissynergistic assets sell them). In addition, in a neighborhood around F^* we also have FSE, so three equilibria (APE, EPE, and FSE) can be sustained. If we fix synergies such

unsustainable. Due to the certainty effect, the information asymmetry of equity becomes second-order, and so $(L, \underline{k})$ will sell assets.

[15] In particular, if $\underline{k} = 0$, all asset sales reduce total surplus since there are no dissynergies. Equity issuance does not affect real efficiency as it leads to a pure wealth transfer between investors and firms; asset sales affect real efficiency due to the difference between common and private values.

[16] Note that, when the conditions in part (i) are satisfied, PSE is unsustainable for any F.

that $1 + \overline{k} > \frac{A_H}{A_L}$, as F rises, we move from a PSE where all H-firms sell assets, to a FSE, then to EPE, and finally to a PSE where all H-firms sell equity. The change in equilibrium as F changes illustrates that H prefers assets (equity) if F is low (high). In addition, it shows how firm boundaries are affected by financing needs.

If we fix F and increase synergies in absolute terms, we move from a PE to a PSE and finally to a FSE. Eisfeldt and Rampini (2006) present a model showing that operational motives for asset sales are procyclical, and empirically find that asset sales are indeed procyclical. This procyclicality may arise not only because operational motives rise in booms, but also because L is able to camouflage asset sales as being operationally-motivated in booms. In our model, an increase in operational motives corresponds to $\underline{k}$ becoming more negative. Specifically, if F is high, with high $\underline{k}$ (weak dissynergies) we have an EPE in which there is no camouflage effect. As $\underline{k}$ falls, we move to an SE where camouflage is possible, since some H-firms are selling assets due to operational reasons. Markets in which H sells assets due to negative k are deep, similar to the notion of "market depth" in Kyle (1985). The liquidity traders in Kyle are analogous to high-quality asset sellers: they are selling assets for reasons other than them having a low common value. The presence of such traders allows informed speculators, who do have assets with a low common value, to profit by selling them.[17]

Note that the regions in Proposition 1 do not overlap.[18] This is because the Proposition gives sufficient conditions for the equilibria to exist, which are not necessary. The proof of Proposition 1 shows that the necessary conditions do overlap, i.e. there is no set of parameters for which all necessary conditions are violated.

3 Negative Correlation

We now turn to the case of negative correlation. Since $A_L > A_H$, we now use the term "high (low)-quality assets" to refer to the assets of L (H). Note that negative correlation is a mild condition: it only means that high-quality firms are not universally high-quality, as they may have some low-quality assets. It does not require the values of the divisions to covary negatively with each other (e.g. that a market upswing

[17] Grenadier, Malenko, and Strebulaev (2012) have a similar notion of camouflage, which they dub "blending in with the crowd," in a different setting. If an industry-wide shock forces good managers to abandon their projects for exogenous reasons, bad managers take the opportunity to abandon their bad projects without their low quality being revealed.

[18] $\frac{\mathbb{E}[A]}{A_L}$ (the upper bound on $1 + \overline{k}$ for APE) is less than $\frac{A_H}{A_L}$ (the lower bound on $1 + \overline{k}$ for PSE) and $\frac{\mathbb{E}[A]}{A_H}\frac{E_H}{E_L}$ (the upper bound on $1+\overline{k}$ for PSE) is less than $\frac{E_H}{E_L}$ (the lower bound on $1+\overline{k}$ for FSE), and similarly for the bound on $1 + \underline{k}$.

helps one division and hurts the other). It is reasonable for the market to know the correlation of the asset with the core business (even if it does not observe quality) simply by observing the type of asset traded. For example, the value of airplanes in a bank's leasing division is unlikely to be highly correlated with the bank as a whole, but the value of mortgages will be.

In this section, the manager's objective function places weight ω on the firm's stock price and $1-\omega$ on fundamental value.[19] We introduce stock price concerns because, with negative correlation, there is now a trade-off involved in selling assets: being inferred as H maximizes the market value, but being inferred as L maximizes proceeds and thus fundamental value. Thus, without stock price concerns, no PE is sustainable.

3.1 Pooling Equilibrium, All Firms Sell Assets

As in Section 2.1, we consider an APE, supported by the OEPB that an equity issuer is of type $\left(L,\overline{k}\right)$. As before, sold assets are valued at $\mathbb{E}\left[A\right]=\pi A_H+(1-\pi)A_L$ and issued equity is valued at E_L. An asset seller has a stock price of $\mathbb{E}\left[C+A\right]-F\times\mathbb{E}\left[k\right]$ and an equity issuer is priced at C_L+A_L. The stock price of an asset seller takes into account the expected loss of synergies from the sale, $\mathbb{E}[k]=\frac{1}{2}\left(\overline{k}+\underline{k}\right)$.

By deviating, L avoids the capital loss from selling highly-valued assets at a pooled price as well as any loss of synergies, but suffers a low stock price. Thus, he will only cooperate if his concern for the stock price ω is high. Since $\left(L,\overline{k}\right)$ is most likely to deviate, all L-firms will cooperate if:

$$\omega\geq\omega^{APE,ND,L}=\frac{F\left(\frac{A_L}{\mathbb{E}[A]}(1+\overline{k})-1\right)}{\pi((C_H-C_L)-(A_L-A_H))-\frac{1}{2}F(\overline{k}+\underline{k})+F\left(\frac{A_L}{\mathbb{E}[A]}(1+\overline{k})-1\right)}. \tag{21}$$

If (21) holds, it is automatic that all H-firms will not deviate: their incentives to deviate are weaker as they are making a capital gain by selling low-quality assets. Thus, (21) is necessary and sufficient for no firms to deviate.

The lower bound given by (21) is relatively loose. It is easy to rule out a deviation to equity. Issuing equity not only leads to a low price (of C_L+A_L) on the equity being sold (as in MM), but also implies a low valuation (of C_L+A_L) for the rest of the firm. This is because the equity being sold is necessarily perfectly correlated with the firm.

[19] The manager's stock price concerns can stem from a number of sources introduced in earlier work, such as takeover threat (Stein (1988)), concern for managerial reputation (Narayanan (1985), Scharfstein and Stein (1990)), or the manager expecting to sell his shares before fundamental value is realized (Stein (1989)).

The second effect is absent in MM, since the manager only cares about fundamental value and not the stock price.

The IC is trivially satisfied. Type $\left(L, \overline{k}\right)$ will indeed deviate to equity if revealed H: his stock price will rise, he will receive a capital gain by selling equity for a high price (compared to his current loss on assets) and he avoids the loss of synergies $\overline{k} \geq 0$.

The results of this subsection are summarized in Lemma 4 below:

Lemma 4. *(Negative correlation, pooling equilibrium, all firms sell assets.) Consider a pooling equilibrium where all firms sell assets ($X_H = X_L = A$) and a firm that sells equity is inferred as type $\left(L, \overline{k}\right)$. The prices of assets and equity are $\pi A_H + (1-\pi) A_L$ and E_L respectively. The stock prices of asset sellers and equity issuers are $\mathbb{E}\left[C + A\right] - F \times \mathbb{E}\left[k\right]$ and $C_L + A_L$, respectively. This equilibrium is sustainable if*

$$\omega \geq \omega^{APE,ND,L} = \frac{F\left(\frac{A_L}{\mathbb{E}[A]}(1+\overline{k}) - 1\right)}{\pi((C_H - C_L) - (A_L - A_H)) - \frac{1}{2}F(\overline{k} + \underline{k}) + F\left(\frac{A_L}{\mathbb{E}[A]}(1+\overline{k}) - 1\right)}.$$

The bound is increasing in F, so again the choice of financing depends on the amount required. However, F plays a different role here than in the positive correlation model, where it drove the certainty effect. Here, a greater F means that L's capital loss from pooling is sustained over a larger base, increasing the fundamental value motive and requiring higher stock price concerns ω to maintain indifference. Put differently, if F is high, L suffers such a large capital loss from selling assets that it prefers to "bite the bullet" and issue equity despite the low stock price.

3.2 Pooling Equilibrium, All Firms Sell Equity

We next consider a PE in which all firms sell equity, supported by the OEPB that an asset seller is of type $(L, \underline{k})$.[20] As before, issued equity is valued at $\mathbb{E}[C + A] + F$

[20]For all equilibria, we specify the OEPB that a deviator is of quality L, and are free to choose whichever k makes the equilibrium most likely to hold. In all equilibria considered thus far, k only affected the IC and so the choice was straightforward: we choose the k that makes the IC easier to satisfy. Thus, for example, the OEPB for APE under positive correlation was that a deviator is $\left(L, \overline{k}\right)$ rather than $(L, \underline{k})$. Here, k affects both IC and ND and so the choice is not straightforward. A lower k makes IC easier to satisfy (as $(L, \underline{k})$ is more willing to deviate to asset sales to get rid of a dissynergistic asset) but increases the stock price of a deviator (as it is deemed to be losing a dissynergistic asset) and makes ND harder to satisfy. We follow the earlier equilibria and choose the k that makes the IC easiest to satisfy. This is because the goal of this section is to show that APE is sustainable for a greater range of parameters than EPE. In Section 3.3 we show that the IC condition for EPE is tighter than the ND condition for APE, so if we chose a different k (which would make the IC condition for EPE harder to satisfy), this would still hold.

and sold assets are valued at A_L. The stock price is $\mathbb{E}[C+A]$ for an equity issuer and $C_L + A_L - F\underline{k}$ for an asset seller.

By deviating, H avoids the capital loss from equity and gets rid of a dissynergistic asset, but suffers a low stock price from being inferred as L. Since $(H, \underline{k})$ is most likely to deviate, all H-firms will cooperate if:

$$\omega \geq \omega^{EPE,ND,H} = \frac{F\left(\frac{E_H}{\mathbb{E}[E]} - \frac{A_H(1+\underline{k})}{A_L}\right)}{\pi((C_H - C_L) - (A_L - A_H)) + F\underline{k} + F\left(\frac{E_H}{\mathbb{E}[E]} - \frac{A_H(1+\underline{k})}{A_L}\right)}. \quad (22)$$

If (22) holds, it is automatic that all L-firms will not deviate: their incentives to deviate are weaker as they are making a capital gain by issuing equity. Thus, (22) is necessary and sufficient for all firms not to deviate. Comparing (22) with (21), the ND condition in APE, the EPE condition is harder to satisfy. In APE, deviation to equity leads to a low price of $C_L + A_L$ not only on the equity sold, but also on the rest of the firm. Here, deviation to asset sales leads to a low price of $C_L + A_L - F\underline{k}$ on the firm, but a high price of A_L on the asset sold, since it is not a carbon copy: the "correlation effect."

Unlike in Section 3.1, the IC is non-trivial, since deviation to asset sales causes $(L, \underline{k})$ to suffer a capital loss. We require:

$$\omega \geq \omega^{EPE,IC} = \frac{F\left(\frac{A_L(1+\underline{k})}{A_H} - \frac{E_L}{\mathbb{E}[E]}\right)}{(1-\pi)((C_H - C_L) - (A_L - A_H)) - F\underline{k} + F\left(\frac{A_L(1+\underline{k})}{A_H} - \frac{E_L}{\mathbb{E}[E]}\right)}. \quad (23)$$

This is also a lower bound. The numerator represents the fundamental loss that L suffers from deviating to asset sales, which arises if the capital loss from selling undervalued assets exceeds the gain from getting rid of a dissynergistic asset. If this loss is positive, he will only deviate if ω is sufficiently high.

The IC condition (23) is stronger than the ND condition (22) if and only if:

$$F(-\underline{k})(N_1 + N_2) < [(C_H - C_L) - (A_L - A_H)][\pi N_2 - (1-\pi)N_1], \quad (24)$$

where N_1 and N_2 are the parenthetical terms in the numerators of (23) and (22), i.e.:

$$N_1 \equiv \frac{A_L(1+\underline{k})}{A_H} - \frac{E_L}{\mathbb{E}[E]} > 0$$
$$N_2 \equiv \frac{E_H}{\mathbb{E}[E]} - \frac{A_H(1+\underline{k})}{A_L} > 0.$$

Lemma 5 below summarizes the equilibrium.

Lemma 5. *(Negative correlation, pooling equilibrium, all firms sell equity.) Consider a pooling equilibrium where all firms sell assets ($X_H = X_L = A$) and a firm that sells assets is inferred as type $(L, \underline{k})$. The prices of assets and equity are given by A_L and $\pi(C_H + A_H) + (1 - \pi)(C_L + A_L) + F$ respectively. The stock prices of asset sellers and equity issuers are $C_L + A_L - F\underline{k}$ and $\mathbb{E}[C + A]$, respectively. This equilibrium is sustainable if $\omega \geq \omega^{EPE}$, where*

$$\omega^{EPE} = \begin{cases} \omega^{EPE,IC} = \dfrac{F\left(\frac{A_L(1+\underline{k})}{A_H} - \frac{E_L}{\mathbb{E}[E]}\right)}{(1-\pi)(C_H - C_L - (A_L - A_H)) + F\left(\underline{k} + \frac{A_L(1+\underline{k})}{A_H} - \frac{E_L}{\mathbb{E}[E]}\right)} & \text{if (24) holds} \\ \omega^{EPE,ND,H} = \dfrac{F\left(\frac{E_H}{\mathbb{E}[E]} - \frac{A_H(1+\underline{k})}{A_L}\right)}{\pi(C_H - C_L - (A_L - A_H)) + F\left(\underline{k} + \frac{E_H}{\mathbb{E}[E]} - \frac{A_H(1+\underline{k})}{A_L}\right)} & \text{if (24) does not hold.} \end{cases} \quad (25)$$

There are two effects of increasing F on the lower bounds. The base effect makes pooling harder to sustain: the capital loss is suffered off a higher base, and so increases H's incentive to deviate. Thus, the lower bound tightens, i.e. increases. This is the same effect as in APE. The second effect is specific to EPE: increasing F reduces the capital loss from pooling, due to the certainty effect, making pooling easier to sustain. The second effect is always smaller, and so the bounds increase overall.

3.3 Comparing the Pooling Equilibria

We now study the conditions under which each PE is sustainable. The goal of the comparison is to show that the correlation effect leads to asset sales being preferred to equity. Since this correlation effect does not depend on synergies, we undertake the comparison for $\overline{k} = \underline{k} = 0$.[21] The results are given in Proposition 2 below:

Proposition 2. *(Negative correlation, comparison of pooling equilibria.) Set $\overline{k} = \underline{k} = 0$. An asset-pooling equilibrium is sustainable if $\omega \geq \omega^{APE,ND,L}$ and an equity-pooling equilibrium is sustainable if $\omega \geq \omega^{EPE}$, where $\omega^{APE,ND,L}$ and ω^{EPE} are given by (21) and (25) respectively, and $\omega^{APE,ND,L} < \omega^{EPE}$. Thus, if:*

(i) $0 < \omega < \omega^{APE,ND,L}$, neither pooling equilibrium is sustainable,

(ii) $\omega^{APE,ND,L} \leq \omega \leq \omega^{EPE}$, only the asset-pooling equilibrium is sustainable,

(iii) $\omega^{EPE} \leq \omega < 1$, both the asset-pooling and equity-pooling equilibria are sustainable.

The thresholds $\omega^{APE,ND,L}$ and ω^{EPE} are both increasing in F.

[21] In contrast, if $\overline{k} >> 0$, trivially APE would be difficult to sustain, despite the correlation effect, as firms will not wish to part with synergistic assets.

Proposition 2 shows that, with negative correlation, asset sales are more common than equity. The range of ω's over which EPE is sustainable is a strict subset of that over which APE is sustainable. This correlation effect is absent in a standard financing model of security issuance, because both debt and equity are positively correlated with firm value. Thus, the issuance of debt may imply that debt is low-quality, and thus the remainder of the firm is also low-quality.

The preference for asset sales analysis points to an interesting benefit of diversification. Stein (1997) notes that an advantage of holding assets that are not perfectly correlated is "winner-picking": a conglomerate can increase investment in the division with the best investment opportunities at the time. Our model suggests that another advantage is "loser-picking": a firm can sell a low-quality asset without implying a low value for the rest of the firm. Non-core assets are a form of financial slack and may even be preferable to debt capacity: debt is typically positively correlated with firm value, so a debt issue may lead the market to infer that both the debt being sold and the remainder of the firm are low-quality.

The analysis also points to a new notion of investment reversibility. Standard theories (e.g. Abel and Eberly (1996)) model reversibility as the real value that can be salvaged by undoing or selling an investment, which in turn depends on the asset's technology. Here, reversibility depends on the market's inference of firm quality if an investment is sold, and thus the correlation between the asset and the rest of the firm.

3.4 Semi-Separating Equilibrium

As in Section 2.3, we have a SE characterized by a cutoff k_q^*. The prices paid for assets and equity are again given by (15) and (16). Since the manager now places weight on the firm's stock price, we need to calculate the stock prices of asset sellers and equity issuers. These are, respectively:

$$\mathbb{E}\left[V|X=A\right] = \pi\left(\frac{k_H^* - \underline{k}}{\mathbb{E}[k_q^*] - \underline{k}}\right)(C_H + A_H) + (1-\pi)\left(\frac{k_L^* - \underline{k}}{\mathbb{E}[k_q^*] - \underline{k}}\right)(C_L + A_L) - \frac{1}{2}F\left(\frac{\mathbb{E}[(k_q^*)^2] - \underline{k}^2}{E[k_q^*] - \underline{k}}\right), \tag{26}$$

$$\mathbb{E}\left[V|X=E\right] = \pi\left(\frac{\overline{k} - k_H^*}{\overline{k} - \mathbb{E}[k_q^*]}\right)(C_H + A_H) + (1-\pi)\left(\frac{\overline{k} - k_L^*}{\overline{k} - \mathbb{E}[k_q^*]}\right)(C_L + A_L). \tag{27}$$

The stock price of an asset seller includes an additional term, $-F \times \mathbb{E}\left[k|X=A\right] = -\frac{1}{2}F\left(\frac{\mathbb{E}[(k_q^*)^2] - \underline{k}^2}{E[k_q^*] - \underline{k}}\right)$, which reflects the expected synergy loss (which may be negative).

Note that $\mathbb{E}[k|X=A] < \mathbb{E}[k]$, since the decision to sell assets suggests that synergies are low. The stock price is higher for an asset seller than an equity issuer ($\mathbb{E}[V|X=A] > \mathbb{E}[V|X=E]$) if and only if:

$$[\Pr(q=H|X=A) - \Pr(q=H|X=E)] \times [(C_H - C_L) - (A_L - A_H)] > F \times \mathbb{E}[k|X=A]. \tag{28}$$

The cutoff k_q^* for a particular quality q is defined by:

$$\omega\left(\mathbb{E}[V|X=A] - \mathbb{E}[V|X=E]\right) = (1-\omega)F\left(\frac{A_q(1+k_q^*)}{\mathbb{E}[A|X=A]} - \frac{C_q + A_q + F}{\mathbb{E}[E|X=E]}\right). \tag{29}$$

Only the parenthetical term on the RHS differs by quality q. Ignoring k, this term will be higher for L, and so $k_H^* > k_L^*$. This is intuitive: since H has low-quality assets but high-quality equity, he is more willing to sell assets. Under positive correlation, $k_H^* > k_L^*$ only if assets exhibit less (certainty effect-adjusted) information asymmetry than equity, as then the capital loss from asset sales is lower. With negative correlation, the capital loss from asset sales is always lower since it is negative (i.e., a capital gain), and so we always have $k_H^* > k_L^*$. From (26) and (27), $k_H^* > k_L^*$ implies that asset (equity) sales lead to a positive (negative) inference about firm quality, i.e. $\Pr(q=H|X=A) > \Pr(q=H|X=E)$, and so the left-hand side ("LHS") of (28) positive. Thus, in the absence of the additional term $F \times \mathbb{E}[k|X=A]$ on the RHS, (28) will hold: the stock price is higher for an asset seller, since H is more likely to sell assets than L. However, if synergies become extremely strong so that $F \times \mathbb{E}[k|X=A]$ is very large, this could theoretically lead to a violation of (28): an asset seller is expected to lose very large synergies, swamping the positive quality inference. Since this paper considers the trade-off between information asymmetry and synergies, to ensure that synergies are not so strong that they dominate the trade-off so that being revealed as low-quality increases the stock price, we assume that (28) holds. A sufficient, but unnecessary, condition is symmetric synergies ($\overline{k} = -\underline{k}$).[22] In turn, (28) implies that the LHS of (29) is positive. Setting $q=H$ on the RHS yields $k_H^* > 0$ for the equality to hold. Intuitively, H will sell assets even if they are moderately synergistic, as he benefits from both the capital gain and the higher stock price.

The amount of financing F has three effects on the cutoffs in (29). To illustrate, consider L's decision. First, an increase in F augments the certainty effect and makes equity less attractive, because L enjoys a smaller capital gain. This tends to increase

[22] In this case, we have $\mathbb{E}[k] = 0$ and so $\mathbb{E}[k|X=A] < \mathbb{E}[k] = 0$; thus, the RHS of (28) is negative and (28) holds.

k_L^*. Second, an increase in F augments fundamental value considerations due to the base effect, decreasing k_L^*. Third, F multiplies the expected synergy loss of an asset seller. If the expected synergy loss $\mathbb{E}[k|X=A]$ is negative (on average, sold assets are dissynergistic), a higher F magnifies this expected gain, increasing the stock price reaction to selling assets and raising k_L^*.

In addition to $k_H^* > k_L^*$ always holding, a second contrast with the positive correlation case is that it is possible to have separation purely by quality and not by synergy, i.e. $k_H^* = \overline{k}$ and $k_L^* = \underline{k}$, where all high (low)-quality firms sell assets (equity). We use SE^q to denote a SE by quality only. Under positive correlation, SE^q is unsustainable as $(L, \underline{k})$ will deviate to assets and enjoy a capital gain plus a loss of dissynergies. Here, such a deviation leads to a capital loss. Indeed, SE^q is sustainable if both of the following conditions hold:

$$\omega \geq \omega^{SE^q,H} = \frac{F\left((1+\overline{k}) - \frac{E_H}{E_L}\right)}{((C_H - C_L) - (A_L - A_H)) - \frac{1}{2}F(\overline{k} + \underline{k}) + F\left((1+\overline{k}) - \frac{E_H}{E_L}\right)} \tag{30}$$

$$\omega \leq \omega^{SE^q,L} = \frac{F\left(\frac{A_L(1+\underline{k})}{A_H} - 1\right)}{((C_H - C_L) - (A_L - A_H)) - \frac{1}{2}F(\overline{k} + \underline{k}) + F\left(\frac{A_L(1+\underline{k})}{A_H} - 1\right)}. \tag{31}$$

The lower bound on ω ensures that stock price concerns deter $\left(H, \overline{k}\right)$ from deviating to retain its synergistic assets.[23] There are three effects of changing F on the lower bound, analogous to the three effects on the cutoffs in (29). First, a rise in F increases $\omega^{SE^q,H}$ due to the certainty effect (reducing $\frac{E_H}{E_L}$). Second, it reduces it due to the base effect. Third, F multiplies the expected synergy loss of an asset seller. If the expected synergy loss $\frac{\overline{k}+\underline{k}}{2}$ is negative, a higher F magnifies this expected gain, reducing $\omega^{SE^q,H}$. The upper bound ensures that $(L, \underline{k})$ will not deviate. If $1 + \underline{k} \leq \frac{A_H}{A_L}$, i.e. the benefits of getting rid of a dissynergistic asset exceed the capital loss from selling high-quality assets, deviation to asset sales yields $(L, \underline{k})$ a fundamental gain and so SE^q can never hold. However, if $1 + \underline{k} > \frac{A_H}{A_L}$, deviation yields $(L, \underline{k})$ a fundamental loss. Since it also leads to a stock price increase, ω must be low to deter deviation. Unlike the lower bound, there are only two effects of changing F on the upper bound as there is no certainty effect. The range of ω's that satisfy (30) and (31) is increasing in $\underline{k}$ and decreasing in $\overline{k}$: the weaker the synergy motive, the easier it is to sustain SE^q.

[23] If $1 + \overline{k} < \frac{E_H}{E_L}$, then the loss of synergies is less than the capital loss that $\left(H, \overline{k}\right)$ would suffer by issuing equity. Thus, H's fundamental value and stock price are both higher under asset sales, and the lower bound on ω is trivially satisfied.

In SE^q, assets are sold for the lowest possible price of A_H and equity is issued at the lowest price of $C_L + A_L$, so there are no capital gains or losses. H's assets are correctly assessed as "lemons," and so the market timing motive for financing (e.g. Baker and Wurgler (2002)) does not exist. However, H is still willing to sell assets despite the lack of a capital gain, due to the positive stock price reaction. Thus, the correlation effect – and its implications for the desirability of financing through asset sales – manifests in two ways. First, APE is sustainable over a greater range of parameters than EPE. Second, SE^q is sustainable, unlike in the positive-correlation model.[24]

Finally, we may have PSEs where one quality pools and the other separates. As in the positive correlation case, if $\overline{k}$ is low, we have a PSE where all H-firms sell assets and L-firms strictly separate. Unlike the positive correlation case, we cannot have a PSE where H-firms issue equity and L-firms strictly separate. Such an equilibrium would require some L-firms to be willing to sell assets but all H-firms not to be. However, since H's assets are lower-quality under negative correlation, H is more willing to sell assets than L. Similarly, if $\underline{k}$ and $\overline{k}$ are high and ω is low, we have a PSE where all L-firms issue equity and H-firms strictly separate. We cannot have a PSE where all L-firms sell assets and H-firms strictly separate: since some H-firms are issuing equity, L-firms will enjoy both a capital gain and a stock price increase by deviating to equity. Thus, the only feasible PSEs involve all H-firms selling assets, or all L-firms issuing equity, which is intuitive since H's assets and L's equity are both low-quality.

The results of this section are summarized in Lemma 6 below.

Lemma 6. *(Negative correlation, semi-separating equilibrium.) Assume that (28) holds.*

(i) A full semi-separating equilibrium is sustainable where quality q sells assets if $k \leq k_q^*$ and equity if $k > k_q^*$, where k_q^* is defined by (29), if $\underline{k}$ is sufficiently low and $\overline{k}$ is sufficiently high. We have $k_H^* > k_L^*$ and $k_H^* > 0$; the sign of k_L^* depends on parameter values. The prices of assets and equity are given by (26) and (27) respectively.

(ii) A partial semi-separating equilibrium in which all firms of quality H (L) sell

[24] SE^q is also featured in the model of Nanda and Narayanan (1999), where core and non-core assets are always negatively correlated and $\omega = 0$. (If these assets are positively correlated, there is no information asymmetry in their model.) Thus, no pooling equilibria are sustainable in the absence of transactions costs. They assume that the transactions costs of asset sales are higher than for equity issuance, which sometimes supports an EPE but never an APE: the opposite result to our paper.

assets (equity) is sustainable if the following two conditions hold:

$$\omega \geq \omega^{SE^q,H} = \frac{F\left((1+\overline{k}) - \frac{E_H}{E_L}\right)}{((C_H - C_L) - (A_L - A_H)) - \frac{1}{2}F(\overline{k} + \underline{k}) + F\left((1+\overline{k}) - \frac{E_H}{E_L}\right)}$$

$$\omega \leq \omega^{SE^q,L} = \frac{F\left(\frac{A_L(1+\underline{k})}{A_H} - 1\right)}{((C_H - C_L) - (A_L - A_H)) - \frac{1}{2}F(\overline{k} + \underline{k}) + F\left(\frac{A_L(1+\underline{k})}{A_H} - 1\right)}.$$

(iii) A partial semi-separating equilibrium where all H-firms sell assets ($k_H^* = \overline{k}$) and L-firms strictly separate ($\underline{k} < k_L^* < \overline{k}$) is sustainable if $\underline{k}$ is sufficiently low, $\overline{k}$ is sufficiently high, and $\omega > \omega^{SE^q,H}$.

(iv) A partial semi-separating equilibrium where all L-firms issue equity ($k_L^* = \underline{k}$) and H-firms strictly separate ($\underline{k} < k_H^* < \overline{k}$) is sustainable if $\underline{k}$ is sufficiently high, $\overline{k}$ is sufficiently high, and $\omega < \omega^{SE^q,L}$.

4 Extensions

This section analyzes extensions of the main model. In Section 4.1, the cash raised is used to finance an investment whose expected value exhibits information asymmetry, and Section 4.2 allows firms to have the choice over whether to raise capital.

In addition, we have undertaken an extension in which firms may also sell the core asset. Since the analysis mainly demonstrates the robustness of the core model, rather than generating new results, we defer it to Appendix B. However, we discuss briefly here two robustness results. First, one of the assets (core or non-core) will exhibit more information asymmetry than the other; since equity is a mix of both assets, its information asymmetry will lie in between. Even though equity is never the safest claim, it may still be issued due to the certainty effect. Second, if the core (non-core) asset is positively (negatively) correlated with firm value, the firm is able to choose the correlation of the asset that it sells, whereas the analysis thus far has considered either the positive correlation case or the negative correlation case. Appendix B shows that a PE in which all firms sell the non-core asset is easier to sustain than one in which all firms sell equity, and one in which all firms sell the core asset. Thus, the correlation effect continues to apply when firms can choose the correlation of the assets they sell.

4.1 Cash Used For Investment

In this section, the cash raised is used to finance an investment whose expected value exhibits information asymmetry. To make the effects of investment as clear as possible, we will focus on the no-synergies case of $\overline{k} = \underline{k} = 0$. We also assume that:

$$\frac{A_H}{A_L} < \frac{C_H + A_H}{\mathbb{E}\left[C + A\right]}. \tag{32}$$

Equation (32) states that the information asymmetry of assets is not too high compared to equity. If (32) is violated, the information asymmetry of assets is so high that, in the core model, EPE is always sustainable regardless of F (the RHS of (12) is negative). We discuss the effect of relaxing (32) at the end of this section.

Since all agents are risk-neutral, only expected values matter. Thus, the model is unchanged if we simply make the investment volatile, so that its payoff is a random variable with an expected value independent of q.[25] For the investment to affect the analysis, it must vary with q so that it exhibits information asymmetry – a critically different concept to volatility. We thus assume that F is used to finance an investment with expected value $R_q = F(1 + r_q)$, where $r_H \geq 0$ and $r_L \geq 0$: since there are no agency problems, only positive-NPV investments are undertaken (as in MM). We allow for both $r_H \geq r_L$ and $r_H < r_L$. The former is more common as high-quality firms typically have superior investment opportunities, but $r_H < r_L$ can occur as a firm that is currently weak may have greater room for improvement.

Intuitively, it would seem that, if $r_H \geq r_L$, the uncertainty of investment will exacerbate the uncertainty of assets in place, weakening the certainty effect and making equity less desirable. However, we will show that this is not necessarily the case. We consider the case of positive correlation here; the case of negative correlation is very similar to the core model and is in Appendix D. Appendix D also allows for $r_H < 0$ and $r_L < 0$ and shows that the core intuitions are unchanged.

We first consider APE. The analog of (8), H's ND condition, is now:

$$\frac{A_H}{\mathbb{E}[A]} \leq \frac{C_H + A_H + F(1 + r_H)}{C_L + A_L + F(1 + r_L)}. \tag{33}$$

As is intuitive, C_q and R_q $(= F(1 + r_q))$ enter symmetrically in all expressions: an equity investor receives a share of C, R, and A, but an asset purchaser receives only a

[25] If the expected value of the investment is F, all of the expressions remain the same. If the expected value is Q, F is simply replaced by Q in all expressions: the relevant variable becomes the (common) expected value of the investment instead of the amount of cash required to finance it.

share of A. From (33), H will not deviate if:

$$F\left[A_H\left(1+r_L\right)-\mathbb{E}\left[A\right]\left(1+r_H\right)\right]\leq\mathbb{E}\left[A\right]\left(C_H+A_H\right)-A_H\left(C_L+A_L\right). \quad (34)$$

Since (32) implies $\frac{A_H}{\mathbb{E}[A]}<\frac{C_H+A_H}{C_L+A_L}$, the RHS of (34) is positive. We first consider the case of $\frac{A_H}{\mathbb{E}[A]}>\frac{1+r_H}{1+r_L}$, i.e. the information asymmetry of investment is not too high. The LHS of (34) is positive, and so we again have an upper bound on F, given by:

$$F\leq\frac{\mathbb{E}\left[A\right]\left(C_H+A_H\right)-A_H\left(C_L+A_L\right)}{A_H\left(1+r_L\right)-\mathbb{E}\left[A\right]\left(1+r_H\right)}. \quad (35)$$

In the core model ((7)), and setting synergies to zero, the denominator is $A_H-\mathbb{E}\left[A\right]$. If $r_L>r_H$, the denominator of (35) is greater than in the core model, and so it is harder to support APE. This is intuitive: L's superior growth options counterbalance its inferior assets in place and reduce the information asymmetry of equity, encouraging deviation. One may think that the reverse intuition applies to $r_H\geq r_L$, but if $\frac{A_H}{\mathbb{E}[A]}>\frac{r_H}{r_L}$, the denominator of (35) is still higher than in the core model. This intuition is incomplete because financing investment has two effects. They can be best seen by the following decomposition of the investment returns:

$$\begin{aligned}R_L&=F\left(1+r_L\right)\\R_H&=F\left(1+r_L\right)+F\left(r_H-r_L\right).\end{aligned}$$

The first, intuitive effect is the $F\left(r_H-r_L\right)$ term which appears in the R_H equation only. The value of investment is greater for H and so it suffers a greater capital loss from selling equity. However, there is a second effect, captured by the $F\left(1+r_L\right)$ term which is common to both qualities. This increases the certainty effect: since the investment is positive-NPV, an equity investor now has a claim to a larger certain value: $F\left(1+r_L\right)$ rather than F. Put differently, while investors do not know firm quality, they do know that the funds they provide will increase in value, regardless of quality. Note that equity does not become attractive to investors simply because the firm is worth more due to its growth opportunities. The growth opportunities are fully priced into the equity issue and are not a "freebie." Instead, the attraction arises because the certain component of the firm's balance sheet is now greater. Due to this second effect, $r_H\geq r_L$ is not sufficient for the upper bound to relax. Only if $\frac{A_H}{\mathbb{E}[A]}<\frac{r_H}{r_L}$ does the first effect dominate, loosening the upper bound. Finally, if $\frac{A_H}{\mathbb{E}[A]}\leq\frac{1+r_H}{1+r_L}$, i.e. investment exhibits high information asymmetry, then the LHS of (34) is non-positive and so the ND condition holds for any F.

Another way to view the intuition is as follows. Equityholders obtain a portfolio of assets in place $(C + A)$ and the new investment (R); F determines the weighting of the new investment in this portfolio. H cooperates if his capital loss from asset sales, $\frac{A_H}{\mathbb{E}[A]}$, is less than the weighted average loss on this overall portfolio. If both the assets in place and the new investment exhibit higher information asymmetry than non-core assets, i.e. $\frac{A_H}{\mathbb{E}[A]} \leq \frac{C_H+A_H}{C_L+A_L}$ and $\frac{A_H}{\mathbb{E}[A]} \leq \frac{1+r_H}{1+r_L}$, then the loss on the portfolio is greater regardless of the weights – hence, H cooperates holds regardless of F. Deviation is only possible if the investment is safer than non-core assets, i.e. $\frac{A_H}{\mathbb{E}[A]} > \frac{1+r_H}{1+r_L}$. In this case, the weight placed on the new investment (F) must be low for the weighted average loss to remain higher for the portfolio, and so for deviation to be ruled out.

Regardless of the specific values of r_H and r_L, in all cases we require F to be below an upper bound.[26] Thus, the result of the core model, that F must be low for APE to be sustainable, continues to hold when cash is used to finance an uncertain investment.

The equilibrium is summarized in Lemma 7 below. The proof of the Lemma shows that the effect of uncertain investment on the IC condition is similar, and that the IC is always stronger than the ND condition (34).

Lemma 7. *(Positive correlation, pooling equilibrium, all firms sell assets, cash used for investment.) Consider a pooling equilibrium where all firms sell assets* $(X = A)$ *and a firm that issues equity is inferred as quality* L*. The prices of assets and equity are* $\pi A_H + (1 - \pi)A_L$ *and* $C_L + A_L + F(1 + r_L)$ *respectively. The equilibrium is sustainable if:*

$$F\left(\mathbb{E}[A](1 + r_L) - A_L(1 + r_H)\right) \leq A_L(C_H + A_H) - \mathbb{E}[A](C_L + A_L). \tag{36}$$

(i) If $\frac{1+r_H}{1+r_L} \geq \frac{\mathbb{E}[A]}{A_L}$*, the asset-pooling equilibrium is sustainable for all* F*.*

(ii) If $\frac{\mathbb{E}[A]}{A_L} > \frac{1+r_H}{1+r_L}$*, the asset-pooling equilibrium is sustainable if* $F \leq F^{APE,IC,I} = \frac{A_L(C_H+A_H)-\mathbb{E}[A](C_L+A_L)}{\mathbb{E}[A](1+r_L)-A_L(1+r_H)}$*. Compared to the case where cash remains on the balance sheet (Lemma 1):*

(a) If $\frac{r_H}{r_L} < \frac{\mathbb{E}[A]}{A_L}$*, the upper bound on* F *is tighter and the asset-pooling equilibrium is sustainable across a smaller range of* F*,*

(b) If $\frac{r_H}{r_L} > \frac{\mathbb{E}[A]}{A_L}$*, the upper bound on* F *is looser and the asset-pooling equilibrium is sustainable across a larger range of* F*.*

The effect of using cash for investment is similar in EPE, so we defer the analysis to Appendix D. The comparison of equilibria is summarized in Proposition 3:

[26] For $\frac{A_H}{\mathbb{E}[A]} \leq \frac{1+r_H}{1+r_L}$, the upper bound is infinite.

Proposition 3. *(Positive correlation, cash used for investment, comparison of equilibria.) An asset-pooling equilibrium is sustainable if $F \leq F^{APE,I}$, and an equity-pooling equilibrium is sustainable if $F \geq F^{EPE,I}$, where $F^{APE,I}$ and $F^{EPE,I}$ are given by:*

$$F^{APE,I} = \begin{cases} \frac{A_L(C_H+A_H)-\mathbb{E}[A](C_L+A_L)}{\mathbb{E}[A](1+r_L)-A_L(1+r_H)} \text{ if } \frac{\mathbb{E}[A]}{A_L} > \frac{1+r_H}{1+r_L}, \\ \infty \text{ if } \frac{1+r_H}{1+r_L} \geq \frac{\mathbb{E}[A]}{A_L}, \end{cases}$$

$$F^{EPE,I} = \begin{cases} \frac{A_L\mathbb{E}[C+A]-A_H(C_L+A_L)}{A_H(1+r_L)-A_L(1+\mathbb{E}[r_q])} \text{ if } \frac{1+r_H}{1+r_L} < \frac{A_H-(1-\pi)A_L}{\pi A_L} \text{ and } \frac{1+r_H}{1+r_L} < \frac{C_H+A_H}{C_L+A_L}; \\ \frac{A_L(C_H+A_H)-A_H\mathbb{E}[C+A]}{A_H(1+\mathbb{E}[r_q])-A_L(1+r_H)} \text{ if } \frac{1+r_H}{1+r_L} < \frac{A_H-(1-\pi)A_L}{\pi A_L} \text{ and } \frac{1+r_H}{1+r_L} \geq \frac{C_H+A_H}{C_L+A_L}; \\ \infty \text{ if } \frac{1+r_H}{1+r_L} \geq \frac{A_H-(1-\pi)A_L}{\pi A_L} \left(> \frac{\mathbb{E}[A]}{A_L}\right). \end{cases}$$

The thresholds $F^{APE,I}$ and $F^{EPE,I}$ are both increasing in r_H and decreasing in r_L. If $\frac{1+r_H}{1+r_L} < \frac{A_H}{\mathbb{E}[A]}$ we have $F^{EPE,I} < F^{APE,I}$. If $\frac{A_H}{\mathbb{E}[A]} < \frac{1+r_H}{1+r_L} < \frac{\mathbb{E}[A]}{A_L}$, we have $F^{EPE,I} < F^{APE,I}$ if and only if $\frac{1+r_H}{1+r_L} < \frac{C_H+A_H}{C_L+A_L}$.

Proposition 3 demonstrates the core model's results continue to hold when there is information asymmetry over the use of the cash raised. Regardless of r_H and r_L, APE (EPE) is sustainable for low (high) F. As in the core model, the source of financing depends on the amount of financing required.

In addition to demonstrating robustness, this extension also generates a new prediction. As r_H falls and r_L rises (the information asymmetry of investment falls), the upper bound on APE tightens and the lower bound on EPE loosens. Thus, the source of financing also depends on the use of financing: if the funds will be used for valuable investment even if the firm is low-quality (r_L is high), they are more likely to be raised from equity. The use of financing also matters in models of moral hazard (uses more likely to be subject to agency problems will be financed by debt rather than equity) or bankruptcy costs (purchases of tangible assets are more likely to financed by debt rather than equity); here it matters in a model of pure adverse selection. In addition, our predictions for the use of equity differ from a moral hazard model. With moral hazard, if cash is to remain on the balance sheet, equity is undesirable due to the agency costs of free cash flow (Jensen (1986)). Here, equity is preferred due to the certainty effect.

Appendix D also considers the case in which (32) does not hold. In this rare case, assets exhibit such high information asymmetry that EPE holds in the core model regardless of F. In this case, *and* if also the information asymmetry of the investment is sufficiently higher than assets, equity issuance is always possible unless the weight on the investment is sufficiently high that the weighted average information asymmetry (of equity and the new investment) is greater than that of assets. Thus, EPE (APE)

now holds for low (high) F. If either one of the above conditions is not met, we return to the core model's result that EPE holds for high F and APE for low F.

4.2 Capital Raising is a Choice

In the core model, firms are forced to raise F. This section gives firms a choice over whether to raise capital. We first allow all firms to have freedom to do nothing, or instead raise capital of F for an investment that returns $F(1+r)$.[27] We continue to consider the case of positive correlation and reintroduce synergies into the model. The possible equilibria are given in Proposition 4 below:

Proposition 4. *(All firms have a choice on whether to raise capital.) If all firms can either raise equity of F, sell assets of F, or do nothing, we have the following equilibria:*

(i) The equilibria in Section 2 continue to hold under the conditions in that Section plus an additional lower bound on $1+r$. For example, a full semi-separating equilibrium where quality q sells assets if $k \leq k_q^*$ and equity if $k > k_q^*$ holds under the conditions of Lemma 3 plus the additional condition $1+r > \frac{E_H}{\mathbb{E}[E|X=E]}$. The additional lower bounds for the other equilibria are given in Appendix A.[28]

(ii) If $1+r \leq \frac{E_H}{E_L}$, we have a semi-separating equilibrium where quality H sells assets if $k \leq k_H^*$ and does nothing if $k > k_H^*$, and quality L sells assets if $k \leq k_L^*$ and issues equity if $k > k_L^*$.

(iia) If $1+r > \frac{A_H(1+\underline{k})}{A_L}$, the cutoff k_H^* is interior and defined by $1+r = \frac{A_H\left(1+k_H^*\right)}{\mathbb{E}[A|X=A]}$. The cutoff k_L^* is defined by $1 = \frac{A_L\left(1+k_L^*\right)}{\mathbb{E}[A|X=A]}$, where $k_L^* > 0$. If $1+r > (\leq) \frac{A_H}{A_L}$, we have $k_H^* > (\leq)\, k_L^*$.

(iib) If $1+r \leq \frac{A_H(1+\underline{k})}{A_L}$, then $k_H^* = \underline{k}$, i.e. all H-firms do nothing, and $k_L^* = 0$.

(iii) If $r = 0$, we have the same semi-separating equilibria in part (ii) except that L-firms with $k > k_L^*$ either issue equity or do nothing.

Part (i) of Proposition 4 shows that the equilibria of the core model are sustainable if the return on investment r is sufficiently high. Intuitively, H is only willing to sustain the losses from raising capital if the capital can be put to a sufficiently productive use.

Part (ii) shows that if r is low, high-quality firms with synergistic assets will not raise capital at all, since the return on investment is insufficient to outweigh the loss of synergies from selling assets or the capital loss from issuing equity. However, high-quality firms with dissynergistic assets will sell them for operational reasons. As before,

[27]Since the implications of $r_H \neq r_L$ have been analyzed in the previous subsection, we set the return on investment to be independent of firm quality here.

[28]The additional lower bounds for the equilibria studied in Appendix C are given in that section.

low-quality firms sell either equity or assets, depending on their level of synergy. There is no camouflage effect with equity: unlike the SE that exists if r is high (part (i)), here equity automatically reveals the firm as L and leads to a price of E_L. In contrast, asset sales may be undertaken either because the asset is low-quality, or because the asset is dissynergistic. This camouflage effect means that asset sales can stem from high- as well as low-quality firms, and so the asset price exceeds A_L. We thus have $k_L^* \geq 0$: low-quality firms prefer assets to equity, due to the camouflage effect.

The SE in part (i) exhibits greater real efficiency than that in part (ii) since all firms are undertaking profitable investment. It is easier to satisfy the condition for part (i) ($1 + r > \frac{E_H}{\mathbb{E}[E|X=E]}$), and harder to satisfy the condition for part (ii) ($1 + r \leq \frac{E_H}{E_L}$), if F is high. A high F has beneficial real consequences: it encourages H to issue equity and invest, because the certainty effect reduces the capital loss from issuing equity.

Part (iii) shows that, if $r = 0$, even low-quality firms have no reason to issue equity: they cannot exploit overvaluation since there is no camouflage, and they are unable to invest the cash raised. Thus, low-quality firms with sufficiently synergistic assets ($k > k_L^*$) are indifferent between selling equity and doing nothing. Indeed, there exists an equilibrium where all L-firms with $k > k_L^*$ do nothing, and so the equity market shuts down: absent an investment opportunity, the only reason to sell equity is if it is low-quality, and so the "no-trade" theorem applies. In contrast, asset sales may be motivated by operational reasons and so the market continues to function. [29]

We next consider the case in which high-quality firms can choose whether to raise financing, but low-quality firms are forced to do so. This is similar to Miller and Rock (1985), where the need to raise financing reveals that a firm's operations are not generating sufficient cash and thus are low-quality. Since some firms are now forced to raise financing, we do not need a profitable investment opportunity to induce them to do so, and so set $r = 0$. The equilibrium is given in Proposition 5 below:

Proposition 5. *(High-quality firms have a choice on whether to raise capital, low-quality firms must raise capital.) Assume $r = 0$. If H-firms can either raise capital of F or do nothing, and L-firms must raise capital of F, we have a semi-separating equilibrium where quality H sells assets if $k \leq k_H^*$ and does nothing if $k > k_H^*$, and quality L sells assets if $k \leq k_L^*$ and issues equity if $k > k_L^*$. The cutoffs k_q^* are defined by $1 = \frac{A_q\left(1+k_q^*\right)}{\mathbb{E}[A|X=A]}$, where $k_H^* < 0 < k_L^*$.*

[29] Note that $\frac{E_H}{E_L} > \frac{E_H}{\mathbb{E}[E|X=E]}$, so for $\frac{E_H}{E_L} > 1 + r > \frac{E_H}{\mathbb{E}[E|X=E]}$, the SEs in parts (i) and (ii) are both sustainable. The first equilibrium is sustainable: since some high-quality firms are selling equity, the equity price is high, which underpins high-quality firms' willingness to sell equity. The second equilibrium is also sustainable: since no high-quality firms are selling equity, the equity price is low, which underpins high-quality firms' reluctance to sell equity.

(a) If $1 > \frac{A_H(1+\underline{k})}{A_L}$*,* k_H^* *is interior.*
(b) If $1 \leq \frac{A_H(1+\underline{k})}{A_L}$*, then* $k_H^* = \underline{k}$*, i.e. all H-firms do nothing, and* $k_L^* = 0$*.*

Proposition 5 shows that the equilibrium of Proposition 4, part (ii), holds in the case in which only low-quality firms must raise capital. This result also illustrates the camouflage effect. Issuing equity immediately reveals a firm as low-quality, since high-quality firms do not issue equity: they are not forced to do so (since they have no capital needs) and will not voluntarily do so (since there is no investment opportunity). By selling assets, low-quality firms can disguise a financing need that is motivated by desperation (it needs to raise capital as it is low-quality) as instead being motivated by operational reasons. Thus, we have $k_L^* > 0$: low-quality firms prefer to raise capital through selling assets, and will do so even if their assets are synergistic.

5 Implications

This section briefly discusses the main implications of the model. Some are empirically testable; a subset have already been tested, while others are as yet unexplored and would be interesting to study in future research. In addition, the model generates other implications that may not be immediately linkable to an empirical test. Note that empirical analysis should focus on asset sales that are primarily financing-motivated.

The first set of empirical implications concerns the determinants of financing choice. One determinant is the amount of financing required: Proposition 1 shows that equity is preferred for high financing needs, because the certainty effect reduces the information asymmetry of equity, while asset sales are preferred for low financing needs. Thus, equity issuances should be larger on average than financing-motivated asset sales. Moreover, the link between the source of financing and the amount required will be stronger where there is less scope for synergies. With low synergies, only pooling equilibria are sustainable and we have the above link. With high synergies, we have a separating equilibrium where some firms use assets (equity) for high (low) financing amounts. Furthermore, with low synergies, firms will issue the same type of claim for a given financing requirement; with high synergies, we should observe greater heterogeneity across firms in financing choices. Estimating the potential for (dis)synergies is difficult. One potential route is to look across the business cycle: Eisfeldt and Rampini (2006) argue that operational motives are stronger in booms. An alternative direction is to compare across industries. For example, in the energy industry, asset sales frequently involve self-contained plants which likely have little scope for synergies. In consumer-facing industries with the potential for cross-selling multiple products to the same customer base, operational motives should be stronger.

A second determinant of financing choice is use of funds. Proposition 3 demonstrates that equity is more likely in two cases. First, it will be used for purposes with less information asymmetry, such as paying debt or dividends. Thus, a firm that is raising financing due to distress (the need to repay debt) is more likely to issue equity. Second, it will be used to finance investment, if the investment will likely be significantly positive-NPV even for low-quality firms (i.e., r_L is high). Along the cross-section, growth firms with good investment opportunities should raise equity. Over the time series, in a strong macroeconomic environment, even low-quality firms will have good investment projects and so equity is again preferred.

A third determinant of financing choice is firm characteristics. Asset sales are preferred for firms with negatively-correlated assets due to the correlation effect. Thus, conglomerates are more likely to sell assets than firms with closely-related divisions.

A second set of empirical implications concerns the market reaction to financing. If $k_H^* > k_L^*$, which always holds in the negative correlation case, and in the positive correlation case under low F, asset sales lead to a positive stock price reaction and equity issuance leads to a negative stock price reaction. Indeed, Jain (1985), Klein (1986), Hite, Owers, and Rogers (1987), and Slovin, Sushka, and Ferraro (1995), among others, find evidence of the former; a long line of empirical research beginning with Asquith and Mullins (1986) documents the latter.

We now move to implications that may be less readily testable. Firms are more willing to sell assets in deep markets where others are selling for operational reasons, providing camouflage. This prediction is harder to test because it is difficult to identify the actual motive for a given asset sale. A more general implication is that there will be multiplier effects: changes in economic conditions that increase operational motives for asset sales will also increase overvaluation-motivated asset sales. The model's implications regarding synergies are also harder to test given the difficulty in estimating synergies. Equity issuers are likely to have synergistic assets, and asset sellers are likely to be parting with dissynergistic ones. High-quality firms are more likely to sell synergistic assets if their financing needs are low, whereas low-quality firms are more likely to do so if their financing needs are high.

6 Conclusion

This paper has studied a firm's choice between financing through asset sales and equity issuance. One relevant consideration is the relative information asymmetry of non-core assets and equity, a natural extension of the MM insight. This paper introduces three additional effects that drive a firm's financing decision. First, investors in an equity issue share in the cash raised, but purchasers of non-core assets do not. Since the value of cash is certain, this mitigates the information asymmetry of equity: the certainty effect. Thus, low (high) financing needs are met through asset (equity) sales. This result is robust to allowing the cash to be used to finance an uncertain investment.

Second, the choice of financing may also depend on operational motives (synergies). A higher financing need pushes high-quality firms towards equity, due to the certainty effect, and reduces the quality and price of assets sold. The synergy motive also allows firms to disguise an asset sale, that is in reality motivated by the asset's low quality, as instead being motivated by operational reasons (dissynergies): the camouflage effect.

Third, a disadvantage of equity issuance is that the market attaches a low valuation not only to the equity being sold, but also to the remainder of the firm, since both are perfectly correlated. This need not be the case for an asset sale, since the asset being sold need not be a carbon copy of the firm. This correlation effect can lead to asset sales being strictly preferred to equity.

The paper suggests a number of avenues for future research. On the empirical side, it gives rise to a number of new predictions, particularly relating to the amount of financing required and the purpose for which funds are raised. On the theoretical side, a number of extensions are possible. One would be to allow for other sources of asset-level capital raising, such as equity carve-outs. Since issuing asset-level debt or equity does not involve a loss of (dis)synergies, a carve-out is equivalent to asset sales in our model if synergies are zero, but it would be interesting to analyze the case in which synergies are non-zero and the firm has a choice between asset sales, carve-outs, and equity issuance. Another restriction is that, in Section 4.2, where firms can choose whether to raise capital, they raise a fixed amount F (as in MM), since there is a single investment opportunity with a known investment requirement of F. An additional extension would be to allow for multiple investment opportunities of different scale, in which case a continuum of amounts will be raised in equilibrium.

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Online Appendix for "Financing Through Asset Sales"

Alex Edmans and William Mann

A Proofs

Proof of Lemma 1

The IC condition (9) is stronger than the ND condition (7) if and only if

$$\frac{(C_H + A_H)A_L(1+\overline{k}) - \mathbb{E}\left[A\right](C_L + A_L)}{\mathbb{E}[A] - A_L(1+\overline{k})} < \frac{(C_H + A_H)\mathbb{E}[A] - (C_L + A_L)A_H(1+\overline{k})}{A_H(1+\overline{k}) - \mathbb{E}[A]}$$

This yields $(1+\overline{k}) < \frac{\mathbb{E}[A]}{\sqrt{A_H A_L}}$.

Proof of Lemma 2

$F^{EPE,IC}$ is greater than $F^{EPE,ND,H}$ if and only if

$$\frac{A_L\mathbb{E}[C+A](1+\underline{k}) - A_H(C_L + A_L)}{A_H - A_L(1+\underline{k})} > \frac{A_L(C_H + A_H) - A_H\mathbb{E}[C+A](1+\underline{k})}{A_H(1+\underline{k}) - A_L}$$

which becomes:

$$1+\underline{k} > \frac{A_H A_L}{\pi A_H^2 + (1-\pi)A_L^2} = \frac{A_H A_L}{\mathbb{E}[A^2]}.$$

Proof of Lemma 3

We start by analyzing the magnitudes of the cutoffs k_H^* and k_L^*; these results apply regardless of whether the SE is full or partial. We then derive conditions under which we have a FSE, or a PSE. From the cutoff equation (18), we have

$$\frac{A_L(1+k_L^*)}{E_L} = \frac{A_H(1+k_H^*)}{E_H} = \frac{\mathbb{E}[A|X=A]}{\mathbb{E}[E|X=E]}.$$

These equations mean that, in any SE, k_L^* and k_H^* obey the following relationship:

$$1+k_H^* = \lambda\left(F\right)\left(1+k_L^*\right), \tag{37}$$

where $\lambda\left(F\right) \equiv \frac{A_L}{A_H}\frac{E_H}{E_L}$ and is decreasing in F. If $F < (>)\, F^*$, then $\lambda > (<)\, 1$ so $k_H^* > (<)\, k_L^*$ from (37). To ascertain the sign of k_H^*, cross-multiplication of (18) shows that $k_H^* > 0$ if and only if

$$E_H\mathbb{E}[A|X=A] > A_H\mathbb{E}[E|X=E]. \tag{38}$$

We start with case (ia), i.e. $F < F^*$. Since $k_H^* > k_L^*$, there is a positive (negative) price reaction to asset (equity) sales, and so $\mathbb{E}[A|X = A] > \mathbb{E}[A]$ and $\mathbb{E}[E|X = E] < \mathbb{E}[E]$. Thus, a sufficient condition for (38) is $E_H\mathbb{E}[A] > A_H\mathbb{E}[E]$. This condition is equivalent to $F < F^*$, the condition required for case (ia) in the first place. Moving to case (ib), $k_H^* < 0$ if and only if (38) is violated. Since $k_H^* < k_L^*$, we now have $\mathbb{E}[A|X = A] < \mathbb{E}[A]$ and $\mathbb{E}[E|X = E] > \mathbb{E}[E]$. Thus, a sufficient condition is $E_H\mathbb{E}[A] < A_H(\mathbb{E}[E])$. This condition is equivalent to $F > F^*$, the condition required for case (ib) in the first place. For case (ic), we have $\lambda(F^*) = 1$, and so $k_H^* = k_L^*$. If both qualities follow the same cutoff strategies, assets and equity are valued at their unconditional expectations. Thus, the quantities on the RHS of (18) are both equal to one, implying that both cutoffs are equal to zero.

We now derive conditions under which FSE exists. We start with part (iia), where $F < F^*$. The ND condition for (H, k_H^*) is $1 + k_H^* = \frac{E_H}{A_H}\frac{\mathbb{E}[A|X=A]}{\mathbb{E}[E|X=E]}$. Given a pair of cutoff rules k_H^* and k_L^*, and associated valuations $\mathbb{E}[A|X = A]$ and $\mathbb{E}[E|X = E]$, for some H-firms to be willing to issue equity, we must have

$$1 + \overline{k} > \frac{E_H}{A_H}\frac{\mathbb{E}[A|X = A]}{\mathbb{E}[E|X = E]}. \tag{39}$$

The RHS is bounded below by $\frac{E_H}{A_H}\frac{\mathbb{E}[A]}{\mathbb{E}[E]}$ (since $k_H^* > k_L^*$, we have $\mathbb{E}[A] < \mathbb{E}[A|X = A]$ and $\mathbb{E}[E] > \mathbb{E}[E|X = E]$) and above by $\frac{E_H}{E_L}$. Thus, a sufficient condition for some H-firms to issue equity is $1 + \overline{k} \geq \frac{E_H}{E_L}$ and a necessary condition is $1 + \overline{k} > \frac{E_H}{A_H}\frac{\mathbb{E}[A]}{\mathbb{E}[E]}$. Intuitively, if F and $\overline{k}$ are too low, the certainty effect is sufficiently weak that the (certainty-adjusted) information asymmetry of equity is so much higher than that of assets, that even the H-firm with greatest synergies (i.e. $(H, \overline{k})$) will sell assets.

We now turn to the indifference condition for (L, k_L^*), which is

$$1 + k_L^* = \frac{E_L}{A_L}\frac{\mathbb{E}[A|X = A]}{\mathbb{E}[E|X = E]}. \tag{40}$$

We use the intermediate value theorem ("IVT") to derive necessary and sufficient conditions for (L, k_L^*) not to deviate. Suppose we specify a candidate pair of cutoffs k_L' and $k_H' \equiv \lambda(F)(1 + k_L') - 1$, where types (q, k_q') sell assets for $k < k_q'$ and issue equity for $k > k_q'$. Assuming $\overline{k} > k_H'$ so that k_H' is feasible, this constitutes an equilibrium if and only if (L, k_L') is indifferent between the two claims. The incentive of (L, k_L') to sell assets is a function continuous in k_L':

$$f(k_L') \equiv \frac{E_L}{\mathbb{E}[E|X=E]} - \frac{A_L(1+k_L')}{\mathbb{E}[A|X=A]}.$$

If $f(k_L') > (<)\, 0$, (L, k_L') will sell assets (equity). Thus, k_L' is an equilibrium cutoff if and only if $f(k_L') = 0$. Our proof strategy is the following: for a given $F < F^*$, we show that (L, k_L') sells assets if $1 + k_L' = \frac{E_L}{A_L}\frac{\mathbb{E}[A]}{\mathbb{E}[E]}$, and equity if $1 + k_L' = \frac{1+\overline{k}}{\lambda(F)}$. (The latter is the highest possible k_L' given that k_L' and k_H' are related by (37), and k_H' is capped at $\overline{k}$.) Then, by the IVT, there exists a k_L^* between these two values of k_L' for which $f(k_L^*) = 0$ and so the firm is indifferent.

To show that (L, k_L') sells assets if $1 + k_L' = \frac{E_L}{A_L}\frac{\mathbb{E}[A]}{\mathbb{E}[E]}$, we use the fact that $F < F^*$ implies $\lambda(F) > 1$ and so $k_H' > k_L'$. We thus have $\mathbb{E}[A|X = A] > \mathbb{E}[A]$ and $\mathbb{E}[E|X = E] < \mathbb{E}[E]$, which yields $f(k_L') > 0$. In addition, $1 + k_L' = \frac{1+\overline{k}}{\lambda(F)}$ yields

$$f(k_L') = \frac{E_L}{\mathbb{E}[E|X=E]} - \frac{A_H(1+\overline{k})}{\mathbb{E}[A|X=A]}\frac{E_L}{E_H},$$

and so $f(k_L') < 0$ holds if and only if $1 + \overline{k} > \frac{E_H}{A_H}\frac{\mathbb{E}[A|X=A]}{\mathbb{E}[E|X=E]}$, which is the same condition as (39). Thus, the sufficient condition for H, $1 + \overline{k} \geq \frac{E_H}{E_L}$, is also sufficient for L, and so is sufficient for the SE to exist.

The analysis for part (iib) ($F > F^*$) is analogous. The ND condition is now

$$1 + \underline{k} < \frac{E_H}{A_H}\frac{\mathbb{E}[A|X=A]}{\mathbb{E}[E|X=E]}. \tag{41}$$

With $F > F^*$ we now have $\mathbb{E}[A] > \mathbb{E}[A|X = A]$ and $\mathbb{E}[E] < \mathbb{E}[E|X = E]$, so now the RHS of (41) is bounded *above* by $\frac{E_H}{A_H}\frac{\mathbb{E}[A]}{\mathbb{E}[E]}$. Thus, a sufficient condition for some H-firms to sell assets is $1 + \underline{k} \leq \frac{A_L}{A_H}$ and a necessary condition is $1 + \underline{k} < \frac{E_H}{A_H}\frac{\mathbb{E}[A]}{\mathbb{E}[E]}$. Intuitively, if F and $\underline{k}$ are too high, the certainty effect is sufficiently strong that the (certainty-adjusted) information asymmetry of equity is so much lower than that of assets, that even the H-firm with greatest dissynergies (i.e. $(H, \underline{k})$) prefers to sell equity.

We now turn to the ND condition for (L, k_L^*), which remains (40), and again use the IVT. We can easily show that (L, k_L') will deviate to equity at $1 + k_L' = \frac{E_L}{A_L}\frac{\mathbb{E}[A]}{\mathbb{E}[E]}$. A sufficient condition for (L, k_L') to deviate to asset sales at $1 + k_L' = \frac{1+\underline{k}}{\lambda(F)}$ is $1 + \underline{k} \leq \frac{A_L}{A_H}$, which is the same as the sufficient condition for H, and so is sufficient for the SE to exist.

We finally turn to PSE. In case (iiia), all H-firms sell assets and L-firms choose an interior cutoff. Assets are priced at $\mathbb{E}[A|X = A] > \mathbb{E}[A]$ and equity is priced at E_L.

The ND condition for H-firms is:

$$1+\overline{k} \leq \frac{\mathbb{E}[A|X=A]}{A_H}\frac{E_H}{E_L} = \lambda(F)\frac{\mathbb{E}[A|X=A]}{A_L}. \tag{42}$$

A sufficient condition for (42) is $1+\overline{k} \leq \frac{E_H}{E_L}\frac{\mathbb{E}[A]}{A_H}$ and a necessary condition is $1+\overline{k} < \frac{E_H}{E_L}$.

The indifference condition for (L, k_L^*) yields

$$1 + k_L^* = \frac{\mathbb{E}[A|X=A]}{A_L}, \tag{43}$$

and so $k_L^* > 0$: since asset sales are met with a positive price reaction (camouflage effect), L is willing to sell them even if they are synergistic. Combining (42) with (43), we have $\frac{\mathbb{E}[A|X=A]}{A_L} < 1+\overline{k} \leq \lambda(F)\frac{\mathbb{E}[A|X=A]}{A_L}$. Since $\lambda(F^*) = 1$ and $\lambda'(F) < 0$, both conditions can be simultaneously satisfied only if $F < F^*$.

Intuitively, $\frac{\mathbb{E}[A|X=A]}{A_L} < 1+\overline{k} \leq \lambda(F)\frac{\mathbb{E}[A|X=A]}{A_L}$ shows that synergies must be so strong that $\left(L, \overline{k}\right)$ eschews the capital gain from selling overvalued assets and chooses to retain synergistic assets. However, synergies cannot be so strong as to induce $\left(H, \overline{k}\right)$ to deviate to equity. These conditions can simultaneously be satisfied because L considers the gain from selling overvalued *assets*, and H considers the loss from selling undervalued *equity*. Since equity exhibits higher information asymmetry, H will not deviate.

Moreover, for (43) to hold, we must have $1+\overline{k} > \frac{\mathbb{E}[A|X=A]}{A_L}$, for which $1+\overline{k} > \frac{A_H}{A_L}$ is a sufficient condition and $1+\overline{k} > \frac{\mathbb{E}[A]}{A_L}$ is a necessary condition. Intuitively, if $\overline{k}$ is sufficiently low, then all Ls would sell assets, even the type with the highest synergies, since they will get a capital gain of $\frac{\mathbb{E}[A|X=A]}{A_L}$ that is greater than the loss of synergies.

Finally, we need to show that a cutoff k_L^* actually exists at which the cutoff type (L, k_L^*) is indifferent between asset sales and equity (at which the equilibrium condition (43) holds). We again employ the IVT. If we specify a cutoff $1 + k_L'$ equal to the necessary lower bound $\frac{\mathbb{E}[A]}{A_L}$ on $1+\overline{k}$, (L, k_L') deviates to asset sales. Meanwhile, if we specify $1 + k_L' = \frac{A_H}{A_L}$, (L, k_L') deviates to equity. Thus, a pair of sufficient conditions for existence of the equilibrium is $1+\overline{k} \geq \frac{A_H}{A_L}$ and $1+\overline{k} \leq \frac{E_H}{E_L}\frac{\mathbb{E}[A]}{A_L}$.

In case (iiib), all H-firms issue equity and L-firms choose an interior cutoff. Assets are priced at A_L and equity is priced at $\mathbb{E}[E|X=E] > \mathbb{E}[E]$. The indifference condition for (L, k_L^*) yields

$$1 + k_L^* = \frac{E_L}{\mathbb{E}[E|X=E]}, \tag{44}$$

and so $k_L^* < 0$. For (44) to hold, we must have $1+\underline{k} < \frac{E_L}{\mathbb{E}[E|X=E]}$, for which $1+\underline{k} \leq \frac{E_L}{E_H}$ is a sufficient condition and $1+\underline{k} < \frac{E_L}{\mathbb{E}[E]}$ is a necessary condition. Intuitively, if

$\underline{k}$ is sufficiently high, then all Ls would sell equity, even the type with the greatest dissynergies, since they will get a capital gain of $\frac{\mathbb{E}[E|X=E]}{E_L}$ that is greater than the avoidance of dissynergies.

The ND condition for H-firms is

$$1+\underline{k} \geq \frac{A_L}{A_H}\frac{E_H}{\mathbb{E}[E|X=E]} = \lambda(F)\frac{E_L}{\mathbb{E}[E|X=E]}. \tag{45}$$

A sufficient condition for (45) is $1+\underline{k} \geq \frac{A_L}{A_H}\frac{E_H}{\mathbb{E}[E]}$ and a necessary condition is $1+\underline{k} > \frac{A_L}{A_H}$. Combining (45) with (44), we have $\lambda(F)\frac{E_L}{\mathbb{E}[E|X=E]} \leq 1+\underline{k} < \frac{E_L}{\mathbb{E}[E|X=E]}$. Since $\lambda(F^*) = 1$ and $\lambda'(F) < 0$, both conditions can be simultaneously satisfied only if $F > F^*$.

Finally, we need to show that a cutoff k_L^* actually exists at which the cutoff type (L, k_L^*) is indifferent given the resulting equilibrium valuations. We again employ the IVT. If we specify a cutoff $1+k_L'$ equal to the necessary upper bound $\frac{E_L}{\mathbb{E}[E]}$ on $1+\underline{k}$, (L, k_L') deviates to equity. Meanwhile, if we specify $1+k_L' = \frac{E_L}{E_H}$, (L, k_L') deviates to asset sales. Thus, a pair of sufficient conditions for existence of the equilibrium is $1+\underline{k} < \frac{E_L}{E_H}$ and $1+\underline{k} > \frac{A_L}{A_H}\frac{E_H}{\mathbb{E}[E]}$.

Proof of Proposition 1

Parts (i), (ia), and (ib) follow from the discussion of the various equilibria in Lemmas 1-3. For (ic), we first prove $F^{EPE,IC} < F^* < F^{APE,IC}$. Suppose $F \leq F^{EPE,IC}$. This means that the IC is violated for EPE, so that $\frac{A_L(1+\underline{k})}{A_H} \geq \frac{E_L}{\mathbb{E}[E]}$. This implies $\frac{A_L}{A_H} > \frac{E_L}{E_H}$ and so $F < F^*$. Thus $F^{EPE,IC} < F^*$. Similarly, suppose $F \geq F^{APE,IC}$. This means that the IC is violated for APE, so that $\frac{E_L}{E_H} \geq \frac{A_L(1+\overline{k})}{\mathbb{E}[A]}$. This implies $\frac{E_L}{E_H} > \frac{A_L}{A_H}$, and so $F > F^*$. Thus $F^{APE,IC} > F^*$.

Next, we prove that $F^* \leq F^{APE,ND,H}$. $F^* \leq F^{APE,ND,H}$ if $F \geq F^{APE,ND,H}$ implies $F \geq F^*$; and the inequality is strict if $F \geq F^{APE,ND,H}$ implies $F > F^*$. Suppose that $F \geq F^{APE,ND,H}$, so that some H-firm would deviate under APE, i.e. $\frac{A_H(1+\overline{k})}{\mathbb{E}[A]} \geq \frac{E_H}{E_L}$. If $1+\overline{k} < \frac{\mathbb{E}[A]}{A_L}$, then $\frac{A_H(1+\overline{k})}{\mathbb{E}[A]} < \frac{A_H}{A_L}$ and thus $F > F^*$. If we only have $1+\overline{k} \leq \frac{\mathbb{E}[A]}{A_L}$, then $\frac{A_H(1+\overline{k})}{\mathbb{E}[A]} \leq \frac{A_H}{A_L}$ and thus $F \geq F^*$. Recall that $1+\overline{k} \leq \frac{\mathbb{E}[A]}{A_L}$ was a necessary condition for APE to be sustainable, from part (i). Thus, $F^* \leq F^{APE,ND,H}$ whenever APE is sustainable, and the inequality is strict except when $1+\overline{k}$ exactly equals $\frac{\mathbb{E}[A]}{A_L}$.

Finally, we prove $F^{EPE,ND,H} \leq F^*$. $F^{EPE,ND,H} \leq F^*$ if $F \leq F^{EPE,ND,H}$ implies $F \leq F^*$, and the inequality is strict if $F \leq F^{EPE,ND,H}$ implies $F < F^*$ Suppose $F \leq F^{EPE,ND,H}$, so that some H-firm weakly prefers to deviate under EPE, i.e. $\frac{E_H}{\mathbb{E}[E]} \geq \frac{A_H(1+\underline{k})}{A_L}$. If $1+\underline{k} > \frac{E_L}{\mathbb{E}[E]}$, then $\frac{E_H}{E_L} > \frac{A_H}{A_L}$ and thus $F < F^*$. If we only have $1+\underline{k} \geq \frac{E_L}{\mathbb{E}[E]}$, then $\frac{E_H}{E_L} \geq \frac{A_H}{A_L}$ and thus $F \leq F^*$. Recall that $1+\underline{k} \geq \frac{E_L}{\mathbb{E}[E]}$ was a necessary condition for EPE to be sustainable, from part (i). Thus, whenever EPE is sustainable, we have $F^{EPE,ND,H} \leq F^*$, and the inequality is strict except when $1+\underline{k}$ exactly equals $\frac{E_L}{\mathbb{E}[E]}$.

Taking these three points together, whenever both PEs are sustainable, $F^{EPE} \leq F^{APE}$. The inequality is strict unless $1+\overline{k} = \frac{\mathbb{E}[A]}{A_L}$ *and* $1+\underline{k} = \frac{E_L}{\mathbb{E}[E]}$.

Points (ii)-(v) also follow from the discussion of the equilibria in Lemmas 1-3.

It now remains to prove that there are no gaps in the necessary conditions for the various equilibria. We first start with the case of $F < F^*$. Then four of the equilibria stated in the Proposition are possible: APE, a PSE where H sells assets, a FSE with $k_H^* > k_L^*$, and EPE. We prove that there is no combination of parameters that simultaneously violates at least one necessary condition for the first three of these equilibria. Therefore, we are unable to rule out all of the first three equilibria (and so we cannot rule out all four equilibria).

One necessary condition for APE is (5), the ND condition for L, which is violated if $1+\overline{k} > \frac{\mathbb{E}[A]}{A_L}$. A second is given by (8), the ND condition for H. This condition is violated if $1+\overline{k} > \frac{\mathbb{E}[A]}{A_H}\frac{E_H}{E_L}$, which implies either $F > F^*$ or $1+\overline{k} > \frac{\mathbb{E}[A]}{A_L}$. The third is the necessary condition implied by (9), the IC. This condition is violated if $1+\overline{k} < \frac{\mathbb{E}[A]}{A_L}\frac{E_L}{E_H}$, but this in turn implies $F > F^*$ (or else the upper bound on $1+\overline{k}$ is less than 1). In sum, to violate at least one of the necessary conditions for APE given $F < F^*$, we require $1+\overline{k} > \frac{\mathbb{E}[A]}{A_L}$.

For PSE where H sells assets, from part (iiia) of the Proposition, we can rule out this equilibrium if $1+\overline{k} < \frac{\mathbb{E}[A]}{A_L}$ or if $1+\overline{k} > \lambda(F)\frac{\mathbb{E}[A|X=A]}{A_L}$ (note that $\lambda(F) > 1$ when $F < F^*$). Thus, we can rule out both APE and PSE by choosing $\overline{k}$ such that $1+\overline{k} > \lambda(F)\frac{\mathbb{E}[A|X=A]}{A_L}$. Finally, from part (iia), we can rule out FSE if $1+\overline{k} \leq \frac{E_H}{A_H}\frac{\mathbb{E}[A]}{\mathbb{E}[E]}$. The RHS is less than the lower bound $\lambda(F)\frac{\mathbb{E}[A|X=A]}{A_L}$ above, so there is no overlapping range of $\overline{k}$ that simultaneously violates at least one necessary condition for each of these three equilibria.

Moving to $F > F^*$, again four equilibria are possible: EPE, a PSE where H sells equity, a FSE with $k_H^* < k_L^*$, and APE. Again, we prove that there is no combination of parameters that simultaneously violates at least one necessary condition for the first three of these equilibria. One necessary condition for EPE is (11), which is violated if $1+\underline{k} < \frac{E_L}{\mathbb{E}[E]}$. A second is given by (12), the ND condition for L. This condition is violated if either $F < F^*$ or $1+\underline{k} < \frac{E_L}{\mathbb{E}[E]}$. The third is (13), the IC, but to violate this requires $1+\overline{k} > \frac{E_L}{\mathbb{E}[E]}\frac{A_H}{A_L}$, which in turn implies $F < F^*$ (or else the lower bound on $1+\overline{k}$ is greater than 1). In sum, to violate at least one of the necessary conditions for EPE given $F > F^*$, we require $1+\overline{k} > \frac{\mathbb{E}[A]}{A_L}$. $1+\underline{k} < \frac{E_L}{\mathbb{E}[E]}$.

For the PSE where H sells equity, from part (iiib), we can rule out this equilibrium if $1+\underline{k} > \frac{E_L}{\mathbb{E}[E]}$ or if $1+\underline{k} < \lambda(F)\frac{E_L}{\mathbb{E}[E]}$ (note that $\lambda(F) < 1$ when $F > F^*$). Thus, we rule out both EPE and PSE by choosing $\underline{k}$ such that $1+\underline{k} < \lambda(F)\frac{E_L}{\mathbb{E}[E]}$. Finally, from part (iib), we can rule out FSE if $1+\underline{k} > \frac{\mathbb{E}[A]}{\mathbb{E}[E]}$. The RHS is greater than the upper bound $\lambda(F)\frac{E_L}{\mathbb{E}[E]}$ above, so there is no overlapping range of $\underline{k}$ that simultaneously violates at least one necessary condition for each of these three equilibria.

Proof of Proposition 2

After the derivation of Lemmas 4 and 5, it only remains to show the ordering $\omega^{APE,ND,L} < \omega^{EPE}$.

First, since synergies are zero, (24) becomes

$$\pi\left(\frac{A_L}{A_H} - \frac{E_L}{\mathbb{E}[E]}\right) > (1-\pi)\left(\frac{E_H}{\mathbb{E}[E]} - \frac{A_H}{A_L}\right)$$

or equivalently

$$\pi\frac{A_L}{A_H} + (1-\pi)\frac{A_H}{A_L} > (1-\pi)\frac{E_H}{\mathbb{E}[E]} + \pi\frac{E_L}{\mathbb{E}[E]}.$$

Since $\pi > \frac{1}{2}$, this inequality holds if the LHS exceeds 2, i.e.

$$A_L^2 + A_H^2 > 2A_H A_L$$
$$(A_L - A_H)^2 > 0.$$

Since (24) holds, we have $\omega^{EPE} = \omega^{EPE,IC}$. We thus need to prove that $\omega^{APE,ND,L} < \omega^{EPE,IC}$. Since $\pi \geq \frac{1}{2}$, it is sufficient to replace $1-\pi$ with π in the denominator of $\omega^{EPE,IC}$ and show that this new quantity is greater than $\omega^{APE,ND,L}$. These expressions only differ in the numerator, and the numerator of the $\omega^{APE,ND,L}$ is smaller if

$$\frac{A_L}{A_H} - \frac{E_L}{\mathbb{E}[E]} > \frac{A_L}{\mathbb{E}[A]} - 1,$$

which holds because $\frac{A_L}{A_H} > \frac{A_L}{\mathbb{E}[A]}$, and $\frac{E_L}{\mathbb{E}[E]} < 1$.

Proof of Lemma 6

To prove (i), the logic is as follows. We seek a pair of cutoffs (k_H^*, k_L^*) for which both types (q, k_q^*) are indifferent between the two financing sources. As before, we use k_q' to denote candidate cutoffs that may not be equilibria, in response to which we will derive the optimal action of the types.

First we show (under certain assumptions) that, given any candidate cutoff k_H', there will be a k_L' at which type (L, k_L') is indifferent, with this value of k_L' implicitly determined as a continuous function of k_H'. Then we consider candidate equilibria such that k'_L is chosen conditional on k'_H in this manner, and we show that there exists a k'_H where (H, k'_H) is indifferent as well. This method will show that an equilibrium exists.

To prove the first statement, we take as given a cutoff $k'_H > 0$, and we employ the IVT as before, showing that for a sufficiently low (high) k'_L, type (L, k'_L) will deviate to assets (equity). He deviates to asset sales if the difference in stock price between an asset seller and an equity issuer is greater than:

$$(1-\omega)F\left(\frac{A_L(1+k'_L)}{\mathbb{E}[A|X=A]} - \frac{E_L}{\mathbb{E}[E|X=E]}\right).$$

Recall that the difference in stock price is positive by assumption. If $1 + k'_L < \frac{E_L}{E_H}\frac{A_H}{A_L}$, then the above expression is negative, and (L, k'_L) will then deviate to asset sales. On the other hand, as we increase $k'_L \to k'_H > 0$, the relative share price reaction falls to a negative value (the difference in posterior probabilities $Pr(q = H|X = A) - Pr(q = H|X = E)$ falls to zero, and the expected synergy loss grows), while the above expression is positive and increasing. Thus, there will be values of k'_L high enough that type (L, k'_L) issues equity rather than sell assets. Note that both of these conclusions hold regardless of the value of k'_H. Thus, applying the IVT, and allowing sufficiently strong dissynergies that $1 + k'_L < \frac{E_L}{E_H}\frac{A_H}{A_L}$ is feasible, we conclude that for any candidate value of k'_H, there is a value of k'_L at which type (L, k'_L) is indifferent between asset sales and equity. Moreover, since there are no discontinuities in the model, the function implicitly determining this value is continuous.

Turning to the second statement, let us consider different candidate values k'_H, and choose k'_L such that (L, k'_L) is indifferent as described above. Type (H, k'_H) will deviate to asset sales if the (positive) stock price reaction to asset sales relative to equity is greater than

$$(1-\omega)F\left(\frac{A_H(1+k'_H)}{\mathbb{E}[A|X=A]} - \frac{E_H}{\mathbb{E}[E|X=E]}\right).$$

This expression is negative if $1 + k'_H < \frac{E_H}{A_H}\frac{\mathbb{E}[A|X=A]}{\mathbb{E}[E|X=E]}$. Since the RHS of this inequality is greater than 1, there will be values $k'_H > 0$ such that type (H, k'_H) deviates to asset sales. On the other hand, the above expression grows without bound in k'_H, while the difference in the stock price reactions to asset sales and equity is bounded above by $(C_H - C_L) - (A_L - A_H) - F\underline{k}$. Thus, after $\overline{k}$ crosses some threshold $\overline{k}^H$, there will be values of k'_H high enough that type (H, k'_H) issues equity rather than sell its highly-synergistic assets. (As described above, k'_L adjusts in both cases such that type (L, k'_L) remains indifferent.) We conclude that with synergies strong enough such that $k > \overline{k}^H$ and $1 + k < \frac{E_L}{E_H}\frac{A_H}{A_L}$ are both feasible, then there will be at least one pair of cutoff values k_q^* at which types (H, k_H^*) and (L, k_L^*) are both indifferent between equity and asset sales, giving rise to the existence of a FSE.

To prove (ii), it suffices to write out the ND conditions for both qualities, solve for ω, and state the bounds in terms of the type with the synergy value that is most likely to issue a different claim.

To prove (iii), first we examine H's ND condition, which is:

$$\omega\Bigg(Pr(q = H|X = A)\left((C_H - C_L) - (A_L - A_H)\right) - F \times \mathbb{E}[k|k < k_q^*]\Bigg)$$
$$> (1-\omega)F\left(\frac{A_H(1+\overline{k})}{\mathbb{E}[A|X = A]} - \frac{E_H}{E_L}\right)$$

This is relatively easy to satisfy, since the relative share price reaction to asset sales (the LHS of the inequality) is positive, and since the fundamental loss to asset sales relative to equity for H (the RHS of the inequality) is negative in the absence of synergies. In general, the condition is that ω be sufficiently high that even managers with the highest level of synergies cooperate with asset sales. To obtain a condition that is sufficient regardless of the equilibrium value of k_L^*, we consider the limiting case $k_L^* \to \overline{k}$ (the strictest possible condition on ω, where all L-firms are issuing equity). Then the bound on ω is

$$\omega \geq \frac{F\left((1+\overline{k} - \frac{E_H}{E_L}\right)}{((C_H - C_L) - (A_L - A_H)) - \frac{1}{2}F(\overline{k} + \underline{k}) + F\left((1+\overline{k} - \frac{E_H}{E_L}\right)}$$

Note that this bound is identical to $\omega^{SE^q,H}$. In this limiting case, we require the same behavior of H as in the SE^q: all H-firms must cooperate with asset sales, which perfectly reveal their quality, while equity would perfectly "reveal" them to be L.

Next, we again apply the IVT to prove existence of an equilibrium. We first seek a candidate cutoff value k_L' at which (L, k_L') will deviate to asset sales, given the price reactions that result from this cutoff. This happens if the (positive) difference in stock price reactions between asset sales and equity is greater than

$$(1-\omega)F\left(\frac{A_L(1+k_L')}{\mathbb{E}[A|X = A]} - 1\right)$$

When $1 + k_L' = \frac{A_H}{A_L}$, the above expression is negative. Thus if $1 + \underline{k}$ is at least this low, there will be an L-firm that deviates to asset sales.

Finally, we must find a candidate cutoff value k_L' at which (L, k_L') will deviate to equity. Clearly, L will do this if k'_L is sufficiently high, and as we have imposed no upper bound on $\overline{k}$, we conclude that for sufficiently high $\overline{k}$ (along with the previously-imposed bounds on ω and $\underline{k}$), there will be values of k'_L such that L deviates to equity, allowing the equilibrium to exist. (Note that the lower bound on ω increases as we raise $\overline{k}$. This does not invalidate the equilibrium, as that lower bound is still strictly less than 1.)

To prove (iv), we first examine the ND condition for L:

$$\omega\left(1 - Pr(q = H|X = E))((C_H - C_L) - (A_L - A_H)) - \frac{1}{2}F(\underline{k} + k^*_H)\right)$$
$$\leq (1-\omega)F\left(\frac{A_L(1+\underline{k})}{A_H} - \frac{E_L}{\mathbb{E}[E|X=E]}\right)$$

Note that this is more difficult to satisfy than the ND condition for H in (ii). This equilibrium requires all L-firms, and some H-firms, to pool on equity despite the negative share price reaction relative to asset sales (whereas the previous equilibrium required pooling on asset sales, which is encouraged by the price reaction). To satisfy this, we require ω to be sufficiently low. Consider the limiting case $k^*_H \to \overline{k}$. If

$$\omega \leq \frac{F\left(\frac{A_L(1+\underline{k})}{A_H} - 1\right)}{((C_H - C_L) - (A_L - A_H)) - \frac{1}{2}F(\overline{k} + \underline{k}) + F\left(\frac{A_L(1+\underline{k})}{A_H} - 1\right)}$$

then all L-firms will cooperate with equity issuance. The bound on ω is identical to $\omega^{SE^q,L}$: In this limiting case, we require the same behavior of L as in SE^q: all L-firms must cooperate with equity issuance even though it perfectly reveals their quality, while asset sales would perfectly "reveal" them to be H.

Note also that we must also have $1 + \underline{k} > \frac{A_H}{A_L}$ for this to be possible, the reverse of the condition that was imposed in (ii) to ensure that some L-firms sell assets.

Given these conditions, we proceed as before. We find candidate cutoffs k'_H at which (H, k'_H) deviates to asset sales and to equity, and then apply the IVT to conclude that an equilibrium cutoff k^*_H exists between them. H will deviate to asset sales if the positive stock price incentive to sell assets is greater than

$$(1-\omega)F\left((1 + k'_H) - \frac{E_H}{\mathbb{E}[E|X=E]}\right)$$

Since $E_H > \mathbb{E}[E|X = E]$, the above expression is negative, and the inequality holds, for any $k'_H \leq 0$.

Finally, H will deviate to equity if the opposite is true:

$$\omega\left(1 - Pr(q = H|X = E))((C_H - C_L) - (A_L - A_H)) - \frac{1}{2}F(\underline{k} + k_H^*)\right)$$
$$\leq (1-\omega)F\left((1 + k_H') - \frac{E_L}{\mathbb{E}[E|X = E]}\right)$$

With no upper bound imposed on synergies, we can choose $\overline{k}$ sufficiently high that there will be values of k_H' satisfying this inequality.

Proof of Lemma 7

The IC condition holds if:

$$F\left(\mathbb{E}[A](1 + r_L) - A_L(1 + r_H)\right) \leq A_L(C_H + A_H) - \mathbb{E}[A](C_L + A_L). \tag{46}$$

The contrast with the core model ((9)) is similar to the ND conditions. If $\frac{1+r_H}{1+r_L} \geq \frac{\mathbb{E}[A]}{A_L}$, (46) holds for all F. If instead $\frac{1+r_H}{1+r_L} < \frac{\mathbb{E}[A]}{A_L} \leq \frac{r_H}{r_L}$, the upper bound on F becomes looser than in the core model since the information asymmetry of the investment increases L's incentives to deviate and be revealed as H, since he will receive a capital gain on the investment value R as well as the core asset value C. However, if $\frac{\mathbb{E}[A]}{A_L} > \frac{r_H}{r_L}$, then the bound becomes tighter. As with the ND condition, this holds if $r_L > r_H$ (as is intuitive) but can also hold even if $r_H \geq r_L$.

We first consider the case of $\frac{1+r_H}{1+r_L} > \frac{\mathbb{E}[A]}{A_L}$. Since $\frac{\mathbb{E}[A]}{A_L} > \frac{A_H}{\mathbb{E}[A]}$, this case implies that the LHS of both (34) and (36) are negative so they are trivially satisfied, and so the upper bound on F is ∞. If $\frac{A_H}{\mathbb{E}[A]} < \frac{1+r_H}{1+r_L} < \frac{\mathbb{E}[A]}{A_L}$, the ND upper bound is ∞ (the LHS of (34) is negative) but the IC upper bound is finite and as stated in the Lemma. Finally, if $\frac{1+r_H}{1+r_L} < \frac{A_H}{\mathbb{E}[A]}$, both upper bounds are nontrivial (less than ∞). From (32), $\frac{A_H}{\mathbb{E}[A]} < \frac{C_H+A_H}{C_L+A_L}$ and so $\frac{1+r_H}{1+r_L} < \frac{C_H+A_H}{C_L+A_L}$. This is a sufficient condition for the IC to be stronger.

Proof of Proposition 3

It only remains to show that $F^{EPE,I} < F^{APE,I}$ (i.e. the equilibria overlap) if and only if $\frac{1+r_H}{1+r_L} < \frac{C_H+A_H}{C_L+A_L}$. When the APE bound is not trivial ($\frac{1+r_H}{1+r_L} < \frac{\mathbb{E}[A]}{A_L}$), the relevant bound is always given $F^{APE,IC}$, as explained in the proof of Lemma 7. We first wish to show that, when the IC bound is also the relevant bound for EPE, $F^{APE,IC,I} > F^{EPE,IC,I}$ where $F^{EPE,IC,I} = \frac{A_L\mathbb{E}[C+A]-A_H(C_L+A_L)}{A_H(1+r_L)-A_L(1+\mathbb{E}[r_q])}$ (see Appendix D). This inequality is equivalent to:

$$(C_H + A_H)(1 + r_L)[\pi A_L \mathbb{E}[A] - A_H A_L + (1-\pi)A_L^2]$$
$$> (C_L + A_L)(1 + r_H)[\pi A_L \mathbb{E}[A] - A_H A_L + (1-\pi)A_L^2]$$

The bracketed term is positive, so the above inequality is equivalent to $\frac{1+r_H}{1+r_L} < \frac{C_H+A_H}{C_L+A_L}$, which holds since the IC bound is the relevant one for EPE.

When ND is the relevant bound for EPE, we need to compare $F^{APE,IC,I}$ and $F^{EPE,ND,I}$. The proof of Lemma 9 will later show that ND is the relevant bound ($F^{EPE,ND,I} > F^{EPE,IC,I}$) when $\frac{1+r_H}{1+r_L} \geq \frac{C_H+A_H}{C_L+A_L}$, and the paragraph above showed that, if $\frac{1+r_H}{1+r_L} \geq \frac{C_H+A_H}{C_L+A_L}$, $F^{EPE,IC,I} > F^{APE,IC,I}$. Thus, $F^{EPE,I} < F^{APE,I}$ if and only if $\frac{1+r_H}{1+r_L} < \frac{C_H+A_H}{C_L+A_L}$.

Proof of Proposition 4

For part (i), we start with SE, which is similar to Proposition 3. L-firms will not deviate to doing nothing, as they are enjoying a fundamental gain and exploiting a desirable investment opportunity. A high-quality equity issuer will not deviate to doing nothing if

$$1 + r \geq \frac{E_H}{\mathbb{E}\left[E|X = E\right]}, \tag{47}$$

i.e. the capital loss from selling undervalued equity is less than the value of the growth opportunity. Similarly, a high-quality asset seller will not deviate if

$$1 + r > \frac{A_H\left(1 + k_H\right)}{\mathbb{E}\left[A|X = A\right]}.$$

Since k_H^* is defined by $\frac{E_H}{\mathbb{E}[E|X=E]} = \frac{A_H\left(1+k_H^*\right)}{\mathbb{E}[A|X=A]}$, we have $\frac{A_H(1+k_H)}{\mathbb{E}[A|X=A]} < \frac{E_H}{\mathbb{E}[E|X=E]}$ for all asset sellers (because $k_H \leq k_H^*$). Thus, (47) is necessary and sufficient for no firm to deviate and is the condition given in the Proposition.

For the APE of Lemma 1, the additional condition is

$$\frac{A_H\left(1 + \overline{k}\right)}{\mathbb{E}\left[A\right]} < 1 + r,$$

where the LHS is the per-dollar loss suffered by type $\left(H, \overline{k}\right)$ the type that loses the most, and the RHS is the per-dollar gain from raising capital. Similarly, for the EPE of Lemma 2, the additional condition is

$$\frac{E_H}{\mathbb{E}\left[E\right]} < 1 + r.$$

For the PSE of Lemma 3, where all H-firms sell assets, the additional condition is

$$\frac{A_H\left(1+\overline{k}\right)}{\mathbb{E}\left[A|X=A\right]} < 1+r,$$

and for the PSE where all L-firms sell assets, the additional condition is

$$\frac{E_H}{\mathbb{E}\left[E|X=E\right]} < 1+r.$$

Turning to part (ii), we start by considering the case of interior cutoffs. The definitions of k_H^* and k_L^* in the Proposition are given by the indifference conditions. Since $1=\frac{A_L\left(1+k_L^*\right)}{\mathbb{E}[A|X=A]}$, we have $k_L^*>0$. L-firms will not deviate to doing nothing, as they are enjoying a (weakly positive) fundamental gain and exploiting a desirable investment opportunity. A H-firm doing nothing will not deviate to equity if

$$1+r<\frac{E_H}{E_L},$$

i.e. the capital loss from selling undervalued equity exceeds the value of the growth opportunity. If the above is satisfied, it is easy to show that a high-quality asset seller will not deviate either to doing nothing or to issuing equity.

Combining $1+r=\frac{A_H\left(1+k_H^*\right)}{\mathbb{E}[A|X=A]}$ and $1=\frac{A_L\left(1+k_L^*\right)}{\mathbb{E}[A|X=A]}$ yields

$$(1+r)\frac{A_L}{A_H}=\frac{1+k_H^*}{1+k_L^*}.$$

When r is high (specifically, $1+r>\frac{A_H}{A_L}$), we have $k_H^*>k_L^*$: H is more willing to sell assets than L because, if it switches to doing nothing, it loses the valuable growth opportunity (whereas L continues to exploit the growth opportunity if it does not sell assets, since it issues equity instead). When $r\leq\frac{A_H}{A_L}-1$, we have $k_H^*\leq k_L^*$: H is less willing to sell assets than L, because they are undervalued. Note that r is bounded above, since $1+r<\frac{E_H}{E_L}$ for this equilibrium to hold. Thus, we have

$$1+r=\frac{1+k_H^*}{1+k_L^*}\frac{A_H}{A_L}$$
$$\frac{E_H}{E_L}>\frac{1+k_H^*}{1+k_L^*}\frac{A_H}{A_L}.$$

If $\frac{E_H}{E_L}<\frac{A_H}{A_L}$ in Proposition 3, we had $k_H^*<k_L^*$; we similarly have $k_H^*<k_L^*$ here. If

$\frac{E_H}{E_L} > \frac{A_H}{A_L}$ in Proposition 3, we had $k_H^* > k_L^*$. However, $k_H^* > k_L^*$ does not necessarily follow here, since it is possible to have $k_H^* < k_L^*$. As is intuitive, giving the firms the option to do nothing makes H relatively less willing to sell assets, as he has the outside option of doing nothing.

Finally, if $1+r < \frac{A_H(1+\underline{k})}{A_L}$, then all H-firms do nothing: we have a boundary cutoff. The investment opportunity is sufficiently unattractive, and dissynergies are sufficiently weak, that no H-firm wishes to sell its high-quality assets for a low price.

For part (iii), the cutoff k_H^* is defined by the synergy level at which H is indifferent between selling assets and doing nothing. We thus have

$$F = F\left(1 + k_H^*\right) \frac{A_H}{\mathbb{E}\left[A\right]}$$

which yields

$$k_H^* = \frac{\mathbb{E}\left[A\right]}{A_H} - 1 < 0.$$

Similarly, we have

$$F = F\left(1 + k_L^*\right) \frac{A_L}{\mathbb{E}\left[A\right]}$$

which yields

$$k_L^* = \frac{\overline{A}}{\mathbb{E}\left[A\right]} - 1 > 0.$$

Proof of Proposition 5

This proof is a special case of the proof of Proposition 4, part (ii), with $r = 0$.

B Selling the Core Asset

This section verifies robustness of the results of the core model to allowing the firm to sell the core asset. For simplicity we consider the case of no synergies, and thus check robustness of the certainty and correlation effects.

B.1 Positive Correlation

In an APE, assets are sold for $\mathbb{E}\left[A\right] = \pi A_H + (1-\pi) A_L$. An issuer of another claim is inferred as L. Thus, the core asset is sold for C_L, and equity is sold at $C_L + A_L + F$.

As in the core model, H's capital loss is $\frac{F(1-\pi)(A_H - A_L)}{\mathbb{E}[A]}$ from pooling on assets and

$\frac{F(C_H - C_L + A_H - A_L)}{C_L + A_L + F}$ from deviating to equity. Thus, H does not deviate to equity if:

$$F \leq \frac{(C_H + A_H)\mathbb{E}[A] - (C_L + A_L)A_H}{A_H - \mathbb{E}[A]}.$$

If it sells the core asset, its capital loss is $\frac{F(C_H - C_L)}{C_L}$. Thus, to prevent deviation to the core asset, we require:

$$\frac{(1-\pi)(A_H - A_L)}{\mathbb{E}[A]} \leq \frac{C_H - C_L}{C_L}.$$

The OEPB that a seller of the core asset is L satisfies the IC if $\frac{C_L}{C_H} \leq \frac{A_L}{\mathbb{E}[A]}$, which is weaker than the above condition.

In an EPE: equity is sold for $\mathbb{E}[C + A] + F = \pi(C_H + A_H) + (1-\pi)(C_L + A_L) + F$. Core assets are sold for C_L, and non-core assets are sold for A_L. Quality H will not deviate to selling the non-core asset if:

$$F \geq \frac{A_L(C_H + A_H) - A_H\mathbb{E}[C + A]}{A_H - A_L},$$

and the OEPB that a seller of the non-core asset is of quality L satisfies the IC if:

$$F \geq \frac{A_L\mathbb{E}[C + A] - A_H[C_L + A_L]}{A_H - A_L}.$$

Analogously, H will not deviate to selling the core asset if:

$$F \geq \frac{C_L(C_H + A_H) - C_H\mathbb{E}[C + A]}{C_H - C_L},$$

and the OEPB that a seller of the core asset is of quality L satisfies the IC if:

$$F \geq \frac{C_L\mathbb{E}[C + A] - C_H[C_L + A_L]}{C_H - C_L}.$$

As expected, H is more likely to deviate to whichever asset (core or non-core) exhibits the least information asymmetry: this will be the tighter lower bound. More interesting is that equity may be sustainable even though it does not exhibit the least information asymmetry (absent the certainty effect). One of the assets (core or non-core) will exhibit more information asymmetry than the other, and so the information asymmetry of equity will lie in between. It may therefore seem (from MM) that the sale of one asset will always dominate equity, since one of the assets will have lower information asymmetry. However, even though equity is never the safest claim, its issuance may still be sustainable, if F is sufficiently large, due to the certainty effect.

Finally, we now consider a core-asset-pooling equilibrium (CPE). The core asset is sold for $\mathbb{E}[C] = \pi C_H + (1-\pi) C_L$, the non-core asset is sold for A_L, and equity is sold at $C_L + A_L + F$. As in the core model, F does not deviate to equity if:

$$F \leq \frac{(C_H + A_H)\mathbb{E}[C] - (C_L + A_L)C_H}{C_H - \mathbb{E}[C]},$$

and he does not deviate to the non-core asset if:

$$\frac{(1-\pi)(C_H - C_L)}{\mathbb{E}[C]} \leq \frac{A_H - A_L}{A_L}.$$

The IC conditions are trivially satisfied.

Comparing CPE and APE, the former is harder to sustain if

$$\frac{(C_H + A_H)\mathbb{E}[C] - (C_L + A_L)C_H}{C_H - \mathbb{E}[C]} < \frac{(C_H + A_H)\mathbb{E}[A] - (C_L + A_L)A_H}{A_H - \mathbb{E}[A]}$$
$$\frac{A_H}{A_L} < \frac{C_H}{C_L}.$$

Thus, as is intuitive, if the core asset exhibits greater information asymmetry, it is more difficult to sustain its sale. This result is a natural extension of MM and is not the main contribution of the paper. More important is that one of the main insights – the certainty effect and thus the importance of F – remains robust to allowing sales of the core asset.

B.2 Negative Correlation

In this extension, the core (non-core) asset is positively (negatively) correlated with firm value. Thus, the firm is able to choose the correlation of the asset that it sells (whereas in the main model, we either have the positive correlation case or the negative correlation case).

In an APE, H will automatically not deviate. L's objective function is

$$\omega E[C + A] + (1-\omega)\left(C_L + A_L + F - F\frac{A_L}{\mathbb{E}[A]}\right).$$

As in the main paper, if he deviates to equity, his objective function is $C_L + A_L$ and so we require

$$\omega \geq \frac{F\left(\frac{A_L - A_H}{\mathbb{E}[A]}\right)}{\pi((C_H - C_L) - (A_L - A_H)) + F\left(\frac{A_L - A_H}{\mathbb{E}[A]}\right)}.$$

If L deviates to selling the core asset, his objective function is also $C_L + A_L$ and so we have the same condition. This is intuitive: regardless of whether he deviates to the core asset or equity, the claim he issues is fairly priced as he is revealed L, and so his objective function ends up the same. The IC condition that a seller of the core asset or equity is of quality L is trivially satisfied.

In an EPE, L will automatically not deviate. H's objective function is

$$\omega \mathbb{E}[C + A] + (1 - \omega)\left(C_H + A_H + F - F\frac{C_H + A_H + F}{\mathbb{E}\left[C + A\right] + F}\right).$$

If he deviates to non-core assets, his objective function becomes:

$$\omega(C_L + A_L) + (1 - \omega)\left(C_H + A_H + F - F\frac{A_H}{A_L}\right),$$

and if he deviates to core assets, his objective function becomes:

$$\omega(C_L + A_L) + (1 - \omega)\left(C_H + A_H + F - F\frac{C_H}{C_L}\right).$$

H will always deviate to non-core assets than core assets, since $\frac{A_H}{A_L} < 1 < \frac{C_H}{C_L}$. Thus, we have the same ND condition as before:

$$\omega \geq \frac{F\left(\frac{C_H + A_H + F}{\mathbb{E}[C+A]+F} - \frac{A_H}{A_L}\right)}{\pi\left((C_H - C_L) - (A_L - A_H)\right) + F\left(\frac{C_H + A_H + F}{\mathbb{E}[C+A]+F} - \frac{A_H}{A_L}\right)}.$$

The IC condition that a seller of the core asset is of quality L is again trivially satisfied; the IC condition that a seller of the non-core asset is of quality L is satisfied if:

$$\omega \geq \frac{F\left(\frac{A_L}{A_H} - \frac{E_L}{\mathbb{E}[E]}\right)}{(1 - \pi)((C_H - C_L) - (A_L - A_H)) + F\left(\frac{A_L}{A_H} - \frac{E_L}{\mathbb{E}[E]}\right)}.$$

In a CPE, L will automatically not deviate. H's objective function is

$$\omega \mathbb{E}\left[C + A\right] + (1 - \omega)\left(C_H + A_H + F - F\frac{C_H}{\mathbb{E}\left[C\right]}\right).$$

If he deviates to equity, his objective function becomes:

$$\omega(C_L + A_L) + (1-\omega)\left(C_H + A_H + F - F\frac{C_H + A_H + F}{C_L + A_L + F}\right),$$

and if he deviates to non-core assets, his objective function becomes:

$$\omega(C_L + A_L) + (1-\omega)\left(C_H + A_H + F - F\frac{A_H}{A_L}\right).$$

He will always prefer to deviate to non-core assets, since $\frac{A_H}{A_L} < 1 < \frac{C_H+A_H+F}{C_L+A_L+F}$. The ND condition is

$$\omega \geq \frac{F\left(\frac{C_H}{\mathbb{E}[C]} - \frac{A_H}{A_L}\right)}{\pi\left((C_H - C_L) - (A_L - A_H)\right) + F\left(\frac{C_H}{\mathbb{E}[C]} - \frac{A_H}{A_L}\right)}.$$

The IC condition that a seller of equity is of quality L is again trivially satisfied. The IC condition that a seller of the non-core asset is of quality L is satisfied if:

$$\omega \geq \frac{F\left(\frac{A_L}{A_H} - \frac{C_L}{\mathbb{E}[C]}\right)}{(1-\pi)((C_H - C_L) - (A_L - A_H)) + F\left(\frac{A_L}{A_H} - \frac{C_L}{\mathbb{E}[C]}\right)}$$

This condition is stronger than the APE lower bound if and only if:

$$\begin{aligned}\pi\left(\frac{A_L}{A_H} - \frac{C_L}{\mathbb{E}[C]}\right) &> (1-\pi)\left(\frac{A_L - A_H}{\mathbb{E}[A]}\right)\\ \pi\left(\frac{C_L}{\mathbb{E}[C]}\right) &< \frac{\pi A_L \mathbb{E}[A] - (1-\pi)\, A_H\,(A_L - A_H)}{A_H \mathbb{E}[A]}\end{aligned}$$

Since $C_L < \mathbb{E}[C]$, it is sufficient that

$$\begin{aligned}\pi A_H \mathbb{E}[A] &< \pi A_L \mathbb{E}[A] - (1-\pi)\, A_H\,(A_L - A_H)\\ 0 &< (A_L - A_H)\,[\pi \mathbb{E}[A] - (1-\pi) A_H]\end{aligned}$$

which is true since $\pi > 1 - \pi$ and $\mathbb{E}[A] > A_H$.

Thus, the APE is easier to sustain than the CPE. This is a simple extension of the camouflage effect of the core model. A deviation from APE to either the core asset or equity is relatively unattractive, since the firm suffers a "lemons" discount on both the security being issued and the rest of the firm as a whole. This is because both the core asset and equity are positively correlated with the value of the firm. In contrast, a deviation from either CPE or EPE to selling the non-core asset is harder to rule out: even if a high price is received for the non-core asset, this does not imply a high valuation for the firm as a whole, and so it is difficult to satisfy the IC.

The SEs are very similar to the core model. As in the core model, there is a SE where H sells non-core assets and L issues equity. There is also a SE where H sells non-core assets and L sells core assets. The conditions for this equilibrium to hold are exactly the same as in the core model. In both equilibria, by deviating, L's stock price increases but his fundamental value falls by $\frac{F(A_L - A_H)}{A_H}$. Regardless of whether L sells equity or core assets in the SE, deviation involves him selling his highly-valued non-core assets and thus suffering the loss. There is no SE where H sells core assets and L sells equity, or when H sells equity or L sells the core asset, since L will mimic H in both cases. The only possible SE is where H sells non-core assets, as L will not wish to mimic him as this will involve selling assets at a fundamental loss.

B.3 A Three-Asset Model

The previous sub-section showed that, in the case of negative correlation, it is easier to sustain an equilibrium in which all firms sell the non-core asset than one in which all firms sell the core asset. While this result is suggestive of the correlation effect, it may also arise from the fact that the non-core asset exhibits less information asymmetry, because $A_L - A_H < C_H - C_L$. If we reversed this assumption, then firm value would be higher for L than H, and so we would have the same model but with reversed notation. Since firm value is higher for L, then L is effectively H. Since A is positively correlated with firm value, A is effectively C and C is effectively A. We will obtain the result that it is easier to sustain CPE than APE, but this would be because C exhibits less information asymmetry rather than C being negatively correlated.

Thus, to allow for both positively and negatively correlated assets, and also for either asset to exhibit higher information asymmetry, we need to move to a 3-asset model. Let the three assets be C, P, and N. Asset C cannot be sold as it is the core asset, but assets P and N can be. Asset P is the positively correlated asset ($P_H \geq P_L$) and asset N is the negatively correlated asset ($N_H \leq N_L$). We allow for both $P_H - P_L > N_L - N_H$ and $P_H - P_L < N_L - N_H$: either asset may exhibit more information asymmetry. We only assume $C_H + P_H + N_H > C_L + P_L + N_L$: the existence of the third asset C means that H has a higher firm value than L, even if N exhibits more information asymmetry than P. Let $A = P + N$ be the total value of the two non-core assets.

A firm can either sell P, N, or equity. In a NPE, all firms sell the negatively-correlated asset and any firm that sells P or equity is inferred as being L. L has the greatest incentive to deviate and his objective function is

$$\omega\left(\mathbb{E}\left[C+A\right]\right)+\left(1-\omega\right)\left(E_L-F\left(\frac{N_L}{\mathbb{E}\left[N\right]}\right)\right).$$

If L deviates to equity (or to P), it becomes

$$\omega\left(C_L+A_L\right)+\left(1-\omega\right)\left(E_L-F\right).$$

The ND condition is:

$$\omega\pi\left(E_H-E_L\right)\geq\left(1-\omega\right)F\left(\frac{N_L}{\mathbb{E}\left[N\right]}-1\right)$$

$$\omega\geq\frac{F\left(\frac{N_L}{\mathbb{E}[N]}-1\right)}{\pi\left(E_H-E_L\right)+F\left(\frac{N_L}{\mathbb{E}[N]}-1\right)}. \quad (48)$$

The IC conditions that L will deviate to P or equity if it were revealed good are trivially satisfied. L would make a capital gain on selling low-quality P or low-quality equity, compared to its capital loss on selling high-quality N, and enjoy a higher stock price.

In a PPE, all firms sell P and any firm that sells N or equity is inferred as being L. H has the greatest incentive to deviate and his objective function is

$$\omega\left(\mathbb{E}\left[C+A\right]\right)+\left(1-\omega\right)\left(E_H-F\left(\frac{P_H}{\mathbb{E}\left[P\right]}\right)\right).$$

H is strictly better off by deviating to N than to equity, as he will make a capital gain on selling low-quality N rather than a capital loss on selling high-quality equity. If H deviates to N, his objective function becomes

$$\omega\left(C_L+A_L\right)+\left(1-\omega\right)\left(E_H-F\left(\frac{N_H}{N_L}\right)\right).$$

Note that $\frac{N_H}{N_L}<1$: due to the correlation effect, H makes a capital gain from selling the non-core asset. The ND condition is:

$$\omega\pi\left(E_H - E_L\right) \geq (1-\omega)\,F\left(\frac{P_H}{\mathbb{E}\left[P\right]} - \frac{N_H}{N_L}\right)$$
$$\omega \geq \frac{F\left(\frac{P_H}{\mathbb{E}[P]} - \frac{N_H}{N_L}\right)}{\pi\left(E_H - E_L\right) + F\left(\frac{P_H}{\mathbb{E}[P]} - \frac{N_H}{N_L}\right)} \tag{49}$$

The IC condition that L would deviate to N if it were revealed good is:

$$\omega \geq \frac{F\left(\frac{N_L}{N_H} - \frac{P_L}{\mathbb{E}[P]}\right)}{(1-\pi)\left(C_H - C_L + A_H - A_L\right) + F\left(\frac{N_L}{N_H} - \frac{P_L}{\mathbb{E}[P]}\right)} \tag{50}$$

and the IC condition that L would deviate to equity if it were revealed good is:

$$\omega \geq \frac{F\left(\frac{E_L}{E_H} - \frac{P_L}{\mathbb{E}[P]}\right)}{(1-\pi)\left(C_H - C_L + A_H - A_L\right) + F\left(\frac{E_L}{E_H} - \frac{P_L}{\mathbb{E}[P]}\right)} \tag{51}$$

Note that if (50) is satisfied, (51) will be trivially satisfied since $\frac{N_L}{N_H} > 1 > \frac{E_L}{E_H}$, so we ignore (51).

The IC condition for PPE ((50)) is stronger than the ND condition for NPE ((48)) if and only if

$$\pi\left(\frac{N_L}{N_H} - \frac{P_L}{\mathbb{E}[P]}\right) > (1-\pi)\left(\frac{N_L}{\mathbb{E}[N]} - 1\right).$$

This always holds, since$\pi > 1-\pi$, and $\frac{N_L}{N_H} > \frac{N_L}{\mathbb{E}[N]}$, and $\frac{P_L}{\mathbb{E}[P]} < 1$. Thus, it is easier to sustain an equilibrium in which all firms sell negatively-correlated assets than one in which all firms sell positively-correlated assets, due to the correlation effect.

C Semi-Separating Equilibria: Additional Material

For the case of positive correlation, Lemma 3 analyzed PSEs where all H-firms issue one claim, and L-firms mix. This section considers the opposite equilibria where all L-firms issue one claim, and H-firms mix.

Lemma 8. *(Positive correlation, partial semi-separating equilibria where L-firms pool):*

(i) If $F < F^*$, a SE where all L-firms sell equity ($k_L^* = \underline{k}$) and H-firms strictly separate ($\overline{k} < k_H^* < \overline{k}$) is sustainable if $\underline{k} = 0$, $1 + \overline{k} > \frac{E_H}{E_L}$, and π is sufficiently close to

1.

(ii) If $F > F^*$, a SE where all L-firms sell assets ($k_L^* = \overline{k}$) and H-firms strictly separate ($\overline{k} < k_H^* < \overline{k}$) is sustainable if $\overline{k} = 0$, $1 + \underline{k} < \frac{A_L}{A_H}$, and π is sufficiently close to 1.

Proof. In case (i), assets are priced at A_H and equity is priced at $\mathbb{E}[E|X = E] < \mathbb{E}[E]$. The no-deviation condition for L is $\frac{A_L(1+\underline{k})}{A_H} > \frac{E_L}{\mathbb{E}[E|X=E]}$, or equivalently

$$\left(1 + Pr(q = H|X = E)\frac{E_H - E_L}{E_L}\right)(1 + \underline{k}) > 1 + \frac{A_H - A_L}{A_L}$$

Note that $\frac{E_H - E_L}{E_L} > \frac{A_H - A_L}{A_L}$ if and only if $F < F^*$. Then the inequality is satisfied if $F < F^*$, $\underline{k} = 0$ and $Pr(q = H|X = E)$ is sufficiently high. $Pr(q = H|X = E)$ approaches 1 from below as $\pi \to 1$ (in the limit, there are only H types remaining), so for π sufficiently close to 1 we can satisfy the inequality and all L types will cooperate with the equilibrium. It remains to show that there is an equilibrium k_H^* at which type (H, k_H^*) is indifferent between selling assets and issuing equity. As usual, we apply the intermediate value theorem. First we show that there is a candidate value k'_H at which (H, k'_H) deviates to selling assets: this happens if $1 + k'_H < \frac{E_H}{\mathbb{E}[E|X=E]}$, which will be satisfied in particular if $k'_H = 0$. Next we find a candidate k'_H at which (H, k'_H) deviates to issuing equity: this happens if $1 + k'_H > \frac{E_H}{\mathbb{E}[E|X=E]}$. A sufficient condition for such a k'_H to exist is that potential synergies be very high: if $1 + \overline{k} > \frac{E_H}{E_L}$, then we can specify a k'_H that will deviate to equity issuance regardless of the price reaction. Then an equilibrium k_H^* exists between 0 and $\frac{E_H}{E_L}$, allowing the equilibrium to exist. The proof for case (ii) is analogous. Assets are priced at $\mathbb{E}[A|X = A] < E[A]$ and equity is priced at E_H. The no-deviation condition for L is $\frac{A_L(1+\overline{k})}{\mathbb{E}[A|X=A]} < \frac{E_L}{E_H}$, or equivalently

$$\left(1 + \frac{E_H - E_L}{E_L}\right)(1 + \overline{k}) < 1 + Pr(q = H|X = A)\left(\frac{A_H - A_L}{A_L}\right)$$

This will be satisfied if $F > F^*$, $\overline{k} = 0$, and π is sufficiently close to 1 so that $Pr(q = H|X = A)$ is also close to 1. It remains to show that an equilibrium k_H^* exists. Type (H, k'_H) will deviate to asset sales if $1 + k'_H < \frac{\mathbb{E}[A|X=A]}{A_H}$, A sufficient condition for such a k'_H to exist is $1 + k'_H < \frac{A_L}{A_H}$. Type (H, k'_H) will deviate to equity issuance if $1 + k'_H > \frac{\mathbb{E}[A|X=A]}{A_H}$, which is satisfied for $k'_H = 0$. Thus all the conditions stated in the lemma are sufficient for the equilibrium to exist. ■

The intuition behind the sustainability of the PSEs where L pools is similar to the other cases. When L pools, he chooses the security with the most information asymmetry, in contrast to H: he thus pools on equity for $F < F^*$ and on asset sales for $F > F^*$. This reflects the desire of L firms to profit through the camouflage effect, by pooling with the actions of some H-firms.

For these equilibria to be sustainable, we require particularly strong synergies in one direction, so that there will some measure of H-firms that choose each action. For example, when $F < F^*$, H-firms are inclined to sell assets ($k_H^* > 0$) as they exhibit less information asymmetry than equity. However, if $\overline{k}$ is sufficiently high (note that the sufficient condition is higher than that for any other equilibrium), then some H-firms have such strong synergies that the synergy motive swamps information asymmetry conditions so that they retain their assets and issue equity. Once this measure of H-firms is issuing equity, L can pool with them and benefit through the camouflage effect. Note that, by deviating to asset sales, L will be inferred as H, as only H-firms are selling assets in this equilibrium. We thus also require dissynergies $\underline{k}$ to be weak, otherwise some L-firms have operational as well as capital gains motives for selling assets, and will deviate.

When $F > F^*$, the reverse logic holds throughout. H wishes to issue equity, but with sufficiently strong dissynergies, some H-firms will sell assets instead. This allows L to camouflage themselves by pooling with these H-firms. Furthermore, we require synergies $\overline{k}$ to be weak, otherwise some L-firms will have operational motives for issuing equity in addition to the capital gains motives of being inferred as H, and will deviate.

We finally turn to the analysis of Section 4.2, where capital raising is a choice. Part (i) of Proposition 4 stated that the equilibria in the core model continue to hold when capital raising is a choice, with an additional condition that is a lower bound on r. Here we study whether the PSEs where L pools continue to hold when capital raising is a choice.

The equilibrium in part (i) of Lemma 8, where all L-firms issue equity, requires an additional condition to hold. It is clear that L will not deviate to doing nothing, as L is enjoying a capital gain from raising financing, plus exploiting an investment opportunity. However, H may deviate to doing nothing. The H-firms that are issuing equity will only refrain from doing so if the growth opportunity is sufficiently attractive to outweigh their capital loss, i.e. $\frac{E_H}{\mathbb{E}[E|X=E]} < 1 + r$. Similarly, the H-firms that are selling assets will not do so if $1 + k_H < 1 + r$. Since $1 + k_H \leq \frac{E_H}{\mathbb{E}[E|X=E]}$ for $k \leq k_H^*$, with equality for $k = k_H^*$, $r > k_H^*$ is the additional necessary condition to deter all H-firms from deviating to doing nothing.

However, the equilibrium in part (ii) of Lemma 8, where all L-firms sell assets, does not require any additional condition to hold. As above, it is automatic that no L will deviate to doing nothing. H-firms that are issuing equity will not deviate to doing nothing, since they are making zero capital loss from issuing equity (which is priced at E_H) and exploiting an investment opportunity. The H-firms that are selling assets will be less inclined to deviate to doing nothing than to equity for the same reason: both deviations lead to zero capital loss, but the latter will allow exploitation of the investment opportunity. Thus, the indifference condition $1 + k_H^* = \frac{\mathbb{E}[A|X=A]}{A_H}$ that ensures that quality-H asset sellers do not issue equity also will ensure that they will not do nothing. Essentially, giving the firms the option not to raise capital has no effect: L firms will not exploit this option as they are already enjoying gains from capital raising, and H firms will not exploit this option as they will always prefer to issue equity than do nothing. Thus, no additional condition is required.

D Cash Used For Investment: Additional Material

This section provides additional material relevant to Section 4.1.

D.1 Positive Correlation, Positive-NPV Investment, EPE

We first start by analyzing EPE, for the case of positive correlation and positive-NPV investment. The effect of using cash for investment is similar to the APE case of the core model. Intuitively, it may seem that this usage will always make EPE harder to satisfy because the volatility of the investment reduces the certainty effect. However, if r_H is close to r_L, this volatility effect is outweighed by the fact that the investment is positive-NPV. The equilibrium is given in Lemma 9 below.

Lemma 9. *(Positive correlation, pooling equilibrium, all firms sell equity, cash used for investment.) Consider a pooling equilibrium where all firms sell equity ($X = E$) and an asset seller is inferred as quality L. The prices of assets and equity are A_L and $\pi(C_H + A_H) + (1-\pi)(C_L + A_L) + F$ respectively. This equilibrium is sustainable if*

$$F\left[A_H(1+\mathbb{E}[r_q]) - A_L(1+r_H)\right] \geq A_L(C_H + A_H) - A_H\mathbb{E}[C+A] \quad (52)$$

$$F\left(A_H(1+r_L) - A_L\left(1+\mathbb{E}[r_q]\right)\right) \geq A_L\mathbb{E}[C+A] - A_H(C_L + A_L). \quad (53)$$

where $\mathbb{E}[r_q] = \pi r_H + (1-\pi)r_L$.

(i) If $\frac{1+r_H}{1+r_L} > \frac{A_H - (1-\pi)A_L}{\pi A_L}$, the equity-pooling equilibrium is not sustainable for any F.

(ii) If $\frac{1+r_H}{1+r_L} < \frac{A_H-(1-\pi)A_L}{\pi A_L}$ *and* $\frac{1+r_H}{1+r_L} < \frac{C_H+A_H}{C_L+A_L}$*, the equity-pooling equilibrium is sustainable if* $F \geq F^{EPE,IC,I} = \frac{A_L\mathbb{E}[C+A]-A_H(C_L+A_L)}{A_H(1+r_L)-A_L(1+\mathbb{E}[r_q])}$*. Compared to the case where cash remains on the balance sheet (Lemma 2):*

(a) If $\frac{\mathbb{E}[r_q]}{r_L} < \frac{A_H}{A_L}$*, the lower bound on* F *is looser and the equity-pooling equilibrium is sustainable across a greater range of* F

(b) If $\frac{\mathbb{E}[r_q]}{r_L} \geq \frac{A_H}{A_L}$*, the lower bound on* F *is weakly tighter and the equity-pooling equilibrium is sustainable across a smaller range of* F

(iii) If $\frac{1+r_H}{1+r_L} < \frac{A_H-(1-\pi)A_L}{\pi A_L}$ *and* $\frac{1+r_H}{1+r_L} \geq \frac{C_H+A_H}{C_L+A_L}$*, the equity-pooling equilibrium is sustainable if* $F \geq F^{EPE,ND,I} = \frac{A_L(C_H+A_H)-A_H\mathbb{E}[C+A]}{A_H(1+\mathbb{E}[r_q])-A_L(1+r_H)}$*. Compared to the case where cash remains on the balance sheet (Lemma 2):*

(a) If $\frac{r_H}{\mathbb{E}[r_q]} < \frac{A_H}{A_L}$*, the lower bound on* F *is tighter and the equity-pooling equilibrium is sustainable across a smaller range of* F*,*

(b) If $\frac{r_H}{\mathbb{E}[r_q]} \geq \frac{A_H}{A_L}$*, the lower bound on* F *is weakly looser and the equity-pooling equilibrium is sustainable across a larger range of* F*.*

Proof. We start with the ND condition. By pooling, type H's fundamental value is

$$C_H + A_H + R_H - F\left(\frac{C_H + A_H + R_H}{\mathbb{E}[C+A+R]}\right).$$

By deviating, it becomes:

$$C_H + A_H + R_H - F\left(\frac{A_H}{A_L}\right).$$

Thus, he will not deviate if:

$$F\left[A_H(1+E\left[r_q\right]) - A_L(1+r_H)\right] \geq A_L(C_H + A_H) - A_H\mathbb{E}[C+A]$$

where

$$\mathbb{E}[r_q] = \pi r_H + (1-\pi)r_L.$$

We now move to the IC condition. By pooling, type L's fundamental value is

$$C_L + A_L + R_L - F\left(\frac{C_L + A_L + R_L}{\mathbb{E}[C+A+R]}\right).$$

By deviating to asset sales and being inferred as type H, it becomes:

$$C_L + A_L + R_L - F\left(\frac{A_L}{A_H}\right).$$

Thus, he will deviate if:

$$F\left[A_H(1+r_L) - A_L\left(1+E\left[r_q\right]\right)\right] \geq A_L\mathbb{E}[C+A] - A_H(C_L+A_L).$$

This completes the derivation of conditions (52) and (53), the ND and IC upper bounds respectively. Note that (32) implies that the RHS of both conditions is positive. If $\frac{1+r_H}{1+\mathbb{E}[r_q]} > \frac{A_H}{A_L}$, the LHS of both conditions is negative, so the equilibrium will not be sustainable for any F (the lower bound on F is ∞). If $\frac{1+\mathbb{E}[r_q]}{1+r_L} > \frac{A_H}{A_L} > \frac{1+r_H}{1+\mathbb{E}[r_q]}$, the left side of (52) becomes positive but the LHS of (53) is still negative, so the equilibrium still is not sustainable. Only if $\frac{A_H}{A_L} > \frac{1+\mathbb{E}[r_q]}{1+r_L}$ does it become possible to satisfy both conditions. To facilitate comparison with Lemma 7, this can be rewritten as follows:

$$\frac{A_H}{A_L} > \frac{1+\mathbb{E}[r_q]}{1+r_L} \quad \Longleftrightarrow \quad \frac{1+r_H}{1+r_L} < \frac{\pi A_L + (A_H - A_L)}{\pi A_L}$$

This bound is greater than the corresponding quantity $\frac{\mathbb{E}[A]}{A_L}$ in Lemma 7. To derive the bound $((1+r_H)/(1+r_L)) < ((C_H+A_H)/(C_L+A_L))$ in this case, consider the case in which neither condition is automatically violated, and compare the two conditions: We have $F^{EPE,IC,I} > F^{EPE,ND,I}$ if and only if

$$(C_L+A_L)(1+r_H)(A_HA_L-\pi A_H^2-(1-\pi)A_L^2) > (C_H+A_H)(1+r_L)(A_HA_L-\pi A_H^2-(1-\pi)A_L^2)$$

The common term on both sides is less than zero, so the condition is equivalent to $((1+r_H)/(1+r_L)) < ((C_H+A_H)/(C_L+A_L))$. Thus the lower bound will be as stated in the Lemma, depending on whether this condition is satisfied. ■

D.2 Positive Correlation, Positive-NPV Investment, Relaxing Assumption (32)

We now relax assumption (32), under which assets are not sufficiently volatile that EPE is always sustainable in the core model regardless of F. Assumption (32) was sufficient for the RHS of equations (36), (52), and (53) to be positive. However, if (32) does not hold, i.e. assets exhibit sufficiently high information asymmetry compared to equity, it may be that the RHS of some of these equations becomes negative. In the APE, if the RHS of equation (36) is negative, then if $\frac{\mathbb{E}[A]}{A_L} < \frac{1+r_H}{1+r_L}$, the LHS is positive and so the APE is never sustainable for any F, just as in the core model when (32) is violated. Intuitively, if assets exhibit higher information asymmetry than both equity and the new investment, then no portfolio of equity and the new investment will have greater information asymmetry than assets, and so APE cannot be sustained. However, if $\frac{\mathbb{E}[A]}{A_L} \geq \frac{1+r_H}{1+r_L}$, the LHS is also negative and so we now have a lower bound, $F > \frac{\mathbb{E}[A](C_L+A_L)-A_L(C_H+A_H)}{A_L(1+r_H)-\mathbb{E}[A](1+r_L)}$. If the new investment has high information asymmetry, the portfolio of equity plus the new investment will also have high information asymmetry (allowing APE to hold) if the weight on the new investment is sufficiently high.

Similar intuition applies to the case of EPE. If equity exhibits low information asymmetry *and* investment exhibits low information asymmetry, the portfolio of equity and investment has low information asymmetry (allowing EPE to hold) if the weight on the new investment is sufficiently low. Thus, we now have an upper bound on F.

D.3 Negative Correlation, Positive-NPV Investment

This section considers the negative correlation case of Section 3 where the cash raised is used to finance investment. The results are very similar to the core model. In the absence of synergies, the only semi-separating equilibrium is SE^q, where H sells assets and L issues equity. This equilibrium is unchanged. In the absence of synergies, (30) is satisfied: H has no incentive to deviate as he will suffer a capital loss on undervalued equity and a lower stock price. The ND condition for L is achieved by plugging in $\underline{k} = 0$ into (31):

$$\omega \leq \omega^{SE} = \frac{\frac{F(A_L-A_H)}{A_H}}{\frac{F(A_L-A_H)}{A_H} + (C_H - C_L) - (A_L - A_H)}.$$

The new parameters for the investment return only matter when equity is misvalued, but this deviation condition involves either fairly-valued equity or undervalued assets.

Similarly, for APE, the ND condition for L is unchanged from (21):

$$\omega \geq \omega^{APE,ND} = \frac{F\left(\frac{A_L-A_H}{\mathbb{E}[A]}\right)}{(C_H - C_L) - (A_L - A_H) + F\left(\frac{A_L-A_H}{\mathbb{E}[A]}\right)}.$$

Again, the ND condition involves either fairly-valued equity or undervalued assets, and so is unaffected by the return parameters. As in the core model, it is automatic that H will not deviate, and the intuitive criterion will be satisfied.

The equity-pooling equilibrium does change, and the results are given by Lemma 10 below:

Lemma 10. *(Negative correlation, pooling equilibrium, all firms sell equity, cash used for investment.) Consider a pooling equilibrium where all firms sell assets* ($X_H =$

$X_L = A$) and an equity issuer is inferred as quality L. The prices of assets and equity are $\pi A_H + (1-\pi)A_L$ and $C_L + A_L + F$ respectively. This equilibrium is sustainable if

$$\omega \geq \omega^{EPE,IC,I} = \frac{F\left(\frac{A_L}{A_H} - \frac{C_L+A_L+F(1+r_L)}{\mathbb{E}[C+A]+F(1+\mathbb{E}[r_q])}\right)}{(1-\pi)((C_H - C_L) - (A_L - A_H)) + F\left(\frac{A_L}{A_H} - \frac{C_L+A_L+F(1+r_L)}{\mathbb{E}[C+A]+F(1+\mathbb{E}[r_q])}\right)} \quad (54)$$

where $\mathbb{E}[r_q] = \pi r_H + (1-\pi)r_L$. Compared to the case where cash remains on the balance sheet (Lemma 5):

(i) If $\frac{\mathbb{E}[r_q]}{r_L} < \frac{\mathbb{E}[C+A]+F}{C_L+A_L+F}$, the lower bound on ω is looser and the equity-pooling equilibrium is sustainable across a larger range of ω.

(ii) If $\frac{\mathbb{E}[r_q]}{r_L} \geq \frac{\mathbb{E}[C+A]+F}{C_L+A_L+F}$, the lower bound on ω is tighter and the equity-pooling equilibrium is sustainable across a smaller range of ω;

Proof. As in the core model, it is automatic that L will not deviate. Following similar steps to the core model, H will not deviate if:

$$\omega \geq \frac{F\left(\frac{C_H+A_H+F(1+r_H)}{\mathbb{E}[C+A]+F(1+E[r_q])} - \frac{A_H}{A_L}\right)}{\pi((C_H - C_L) - (A_L - A_H)) + F\left(\frac{C_H+A_H+F(1+r_H)}{\mathbb{E}[C+A]+F(1+E[r_q])} - \frac{A_H}{A_L}\right)}$$

and the IC condition is satisfied if:

$$\omega \geq \frac{F\left(\frac{A_L}{A_H} - \frac{C_L+A_L+F(1+r_L)}{\mathbb{E}[C+A]+F\times\mathbb{E}[1+r_q]}\right)}{(1-\pi)((C_H - C_L) - (A_L - A_H)) + F\left(\frac{A_L}{A_H} - \frac{C_L+A_L+F(1+r_L)}{\mathbb{E}[C+A]+F\times\mathbb{E}[1+r_q]}\right)}$$

Using similar steps to the proof of Proposition 5, the IC condition is stronger than the ND condition. ■

As in Lemma 5, there is a lower bound on ω to ensure that L will be willing to deviate to asset sales if he is inferred as H. Intuitively, it would might that, if $r_H \geq r_L$, using cash for volatile investment would increase the lower bound and make the equilibrium harder to sustain, but this intuition only holds if investment is sufficiently volatile, i.e. $\frac{\mathbb{E}[r_q]}{r_L} > \frac{\mathbb{E}[C+A]+F}{C_L+A_L+F}$ (similar to the results in Section 4.1).

The comparison of equilibria is given by Proposition 6, and is analogous to Proposition 2.

Proposition 6. *(Negative correlation, cash used for investment, comparison of pooling equilibria.) An asset-pooling equilibrium is sustainable if $\omega \geq \omega^{APE,ND,L}$ and an equity-pooling equilibrium is sustainable if $\omega > \omega^{EPE,IC,I}$, where $\omega^{APE,ND}$ and $\omega^{EPE,IC,I}$ are* *given by (21) and (54) respectively and* $\omega^{APE,ND,L} < \omega^{EPE,IC,I}$*. Thus, if:*

(i) $0 < \omega < \omega^{APE,ND}$*, neither pooling equilibrium is sustainable,*

(ii) $\omega^{APE,ND,L} < \omega < \omega^{EPE,IC,I}$*, only the asset-pooling equilibrium is sustainable,*

(iv) $\omega^{EPE,IC} \leq \omega < 1$*, both the asset-pooling and equity-pooling equilibria are sustainable.*

The thresholds $\omega^{APE,ND,L}$ *and* $\omega^{EPE,IC,I}$ *are both increasing in* F*.*

D.4 Negative-NPV Investment

We now consider the case in which the funds raised are used to finance a negative-NPV investment, i.e. there are agency problems. We now specify $-1 < r_L < 0$ and $-1 < r_H < 0$ but, as in the core model, allow for both $r_L \leq r_H$ and $r_L > r_H$.

We start with the positive correlation case, and consider the ND condition in APE, which is

$$F\left[A_H\left(1+r_L\right) - \mathbb{E}\left[A\right]\left(1+r_H\right)\right] \leq \mathbb{E}\left[A\right]\left(C_H + A_H\right) - A_H\left(C_L + A_L\right).$$

As in the core model, this condition is always satisfied if $\frac{1+r_H}{1+r_L} \geq \frac{A_H}{\mathbb{E}[A]}$: if the value of the investment is sufficiently less negative for H, this exacerbates the information asymmetry of the assets in place and makes equity less desirable. For $\frac{1+r_H}{1+r_L} < \frac{A_H}{\mathbb{E}[A]}$, we have a nontrivial upper bound. One might think that, if $r_H < r_L$ (i.e. investment is more value-destructive in H than L), this would mitigates H's superior assets in place and reduce the volatility of equity, making APE harder to sustain. However, this is not necessarily the case: the upper bound on F tightens only under the stronger condition of $\frac{A_H}{\mathbb{E}[A]} < \frac{r_H}{r_L}$ (note that this inequality is in the opposite direction to the positive-NPV case, since r_H and r_L are now negative). As in the positive-NPV case, there are two effects of using cash for negative-NPV investment, which can be seen by the following decomposition of investment returns:

$$R_L = F\left(1+r_L\right)$$
$$R_H = F\left(1+r_L\right) + F\left(r_H - r_L\right).$$

The first, intuitive effect is the $F\left(r_H - r_L\right)$ term which appears in the R_H equation only: if $r_H < r_L$, H's inferior use of invested cash mitigates its superior assets in place and strengthens the certainty effect. However, there is a second effect, captured by the $F\left(1+r_L\right)$ term which is common to both qualities. This weakens the certainty effect: since the investment is negative-NPV, it means that an equity investor now has a claim to a smaller certain value: $F(1+r_L)$ rather than F. Only if $\frac{r_H}{r_L} > \frac{A_H}{\mathbb{E}[A]} > \frac{1+r_H}{1+r_L}$ does the first effect dominate, leading to a decrease (tightening) of the upper bound for APE to be sustained. If $\frac{r_H}{r_L} < \frac{A_H}{\mathbb{E}[A]}$, the upper bound loosens and APE becomes easier to sustain. Equity investors now have a claim to a smaller certain value, which makes equity less attractive.

Turning to the IC condition in APE, the effect of investment now being negative-NPV is analogous to the ND condition. The condition for the bound to be trivially satisfied is exactly the same is in the positive-NPV case: $\frac{1+r_H}{1+r_L} \geq \frac{\mathbb{E}[A]}{A_L}$. If $\frac{1+r_H}{1+r_L} < \frac{\mathbb{E}[A]}{A_L}$, we have a non-trivial upper bound which now tightens if $\frac{r_H}{r_L} > \frac{\mathbb{E}[A]}{A_L} > \frac{1+r_H}{1+r_L}$, whereas it loosened in the positive-NPV case. The lower bound loosens if $\frac{r_H}{r_L} < \frac{\mathbb{E}[A]}{A_L}$. In sum, when cash is used for a negative-NPV investment, APE becomes easier to sustain: since equity investors now have a claim to a smaller certain value, the certainty effect weakens. Only if H destroys sufficiently more value than L does APE become harder to sustain.

The effect on EPE is analogous: the lower bound tightens unless r_H is sufficiently lower than r_L ($r_H < r_L$ is not sufficient). Similarly, for the negative correlation case, the inequalities reverse directions under the case of negative-NPV investment, and the economics are similar to above.